Received: Added at production | Revised: Added at production | Accepted: Added at production

DOI: xxx/xxxx

ARTICLE TYPE

Optimal cut-point selection methods when subclasses are involved under binary classification†

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In practice, we often encounter binary classification problems where both main classes consist of multiple subclasses. For example, in an ovarian cancer study where biomarkers were evaluated for their accuracy of distinguishing non-cancer cases from cancer cases, the non-cancer class consists of healthy subjects and benign cases, while the cancer class consists of subjects at both early and late stages. This paper aims to provides optimal cut-point selection methods for such setting. Furthermore, we also study confidence interval estimation of the estimated optimal cut-points. Simulation studies are carried out to explore the performance of the proposed cut-point selection methods as well as confidence interval estimation methods. A real ovarian cancer data set is analyzed using the proposed methods.

KEYWORDS:
ROC curve, cut-point selection, confidence interval, generalized inference, biomarker evaluation

1 | INTRODUCTION

The ROC curve, a graph of sensitivity against 1-specificity across all possible cut-points, is a commonly used tool for evaluating biomarkers or diagnostic tests under binary classification. The area under ROC curve (*AUC*) is the most widely used accuracy measure for assessing the discriminatory ability of a biomarker or a diagnostic test.[1,2,3,4] In clinical decision-making, it is critical to determine an optimal cut-point of a continuous biomarker or diagnostic test for making diagnostic decisions. There exist several methods for selecting optimal cut-point under binary classification. The Youden index method,[5,6] maximizes the overall accuracy, i.e. the sum of sensitivity and specificity. The northwest corner method[7,8] selects the cut-point corresponding to the shortest distance from the point on ROC curve to the perfection point $(0, 1)$. The maximum area method[9] seeks the cut-point that maximizes the rectangle area under the ROC curve.

In practice, many diseases can be naturally classified into K ($K \geq 3$) stages. For example, generally speaking, Alzeimer's disease (AD) diagnosis results are "non-diseased", "intermediate", and "diseased", among which there exist an inherent ordinal ordering.[10,11] The ROC curve has been extended to ROC surface for trichotomous classification and *AUC* to the volume under the ROC surface (*VUS*), the most popular measure under three classes.[12,13,14] There also exist many research on diagnostic measures developed for general K ($K \geq 3$) classes.[15,16,17] Recently, Hua and Tian[18] studied optimal cut-points selection methods under multi-class classification with ordinal ordering.

Besides ordinal ordering, there exist many special settings where multiple classes are involved. Tree (or umbrella) ordering is a setting which consists of a non-diseased class and a diseased class and the latter consists of several subclasses among which there does not exist a clearly defined ordering. Take lung cancer diagnosis as an example. Biomarkers are evaluated for their accuracy of distinguishing normal tissues from non-small cell lung cancer (NSCLC) which consists of three unordered subtypes

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(adenocarcinoma (ADC), large cell carcinoma (LCC), and squamous-cell carcinoma (SCC)).[19,20,21] Wang et al.[22] proposed an ROC framework under tree (or umbrella) ordering (denoted as TROC), and the area under TROC curve as a bona fide accuracy measure. Furthermore, Wang et al.[23] examined several methods for optimal cut-point selection under tree or umbrella ordering.

Recently, Feng and Tian[24] studied a special setting which can be considered as an extension of tree or umbrella ordering. In this setting, both non-diseased class and diseased class involves multiple subclasses. Such setting is very common in practice. For example, an ovarian cancer study[25] evaluated biomarkers for their discriminatory ability classifying non-cancer cases from cancer cases, where the non-cancer class includes both healthy and benign cases, and the cancer class includes early and late cases. Feng and Tian[24] demonstrated that the commonly adopted pooling strategy can be seriously misleading in biomarker evaluation and further presented an "overall ROC" (ROC_o) curve to evaluate biomarkers. More details will be reviewed in Section 2.

The main goal of this paper is to present a large number of optimal cut-point selection methods under binary classification when subclasses are involved in both main classes. Some of the proposed cut-points selection methods can be viewed as extensions of the methods for tree (or umbrella) ordering[23] utilizing the measures defined in the framework of ROC_o[24], and the rest of methods are based on new ideas requiring definitions of new measures presented in Section 2.2. Additionally, we will also present methods for confidence interval estimation of the proposed optimal cut-points.

The rest of this paper is organized as follows. Section 2.1 reviews the existing definitions of correct classification rates and the "overall ROC" (ROC_o) framework for the settings where subclasses are involved under binary classification, proposed by Feng and Tian[24]. Section 2.2 presents definitions of false classification rates under such setting. Section 3 presents optimal cut-point selection methods including extensions of existing methods (Section 3.1) and new methods (Section 3.2). Section 4 proposes inference methods for confidence interval estimation of the optimal cut-points, including both parametric (Section 4.1 and Section 4.2) and non-parametric methods (Section 4.3). Section 5 presents the simulation results. The proposed optimal cut-point selection methods are illustrated using a subset data from PLCO cancer study in Section 6. Finally, Section 7 contains summary and discussion.

2 | SETTINGS AND DEFINITIONS OF CLASSIFICATION RATES

Consider a binary classification problem with a control class and diseased class as two main classes, and assume the control class and the diseased class include k_1 ($k_1 \geq 1$) and k_2 ($k_2 \geq 1$) subclasses, respectively. Let $k_1 + k_2 = K$. Let $Y_1, \ldots, Y_{k_1}$ stand for variables of biomarker values of subclasses $1, \ldots, k_1$ in control class, respectively, and let $Y_{k_1+1}, \ldots, Y_K$ standard for diseased subclasses $k_1 + 1, \ldots, K$, respectively. Let F_{Y_k} denote the cumulative distribution function for Y_k where $k = 1, \ldots, K$. Biomarkers or diagnostic tests are evaluated for their performance of distinguishing control class from diseased class without requiring specification of an ordering in terms of marker values for subclasses relative to each other within each main class.

2.1 | True classification rates

In the following, we briefly review the definition of true classification rates in "overall ROC" (ROC_o) framework proposed by Feng and Tian.[24]

Define $X = \max(Y_1, \ldots, Y_{k_1})$ and $Z = \min(Y_{k_1+1}, \ldots, Y_K)$. At a given cut-point c, the "overall sensitivity" (Sen_o), i.e. the probability that a randomly selected subject from any diseased subclasses can be correctly identified as diseased, is defined as

$$Sen_o(c) = P(Z > c) = \prod_{j=k_1+1}^{K} (1 - F_{Y_j}(c)), \tag{1}$$

and the "overall specificity" (Spe_o), i.e. the probability that a randomly selected subject from any sub-classes in control group can be correctly classified as control, is defined as

$$Spe_o(c) = P(X \leq c) = \prod_{i=1}^{k_1} F_{Y_i}(c). \tag{2}$$

For $c \in (-\infty, \infty)$, an overall ROC (ROC_o) curve[24] was defined as

$$ROC_o(F_1, \ldots, F_{k_1}, \ldots, F_K) = (1 - Spe_o(c), Sen_o(c)).$$

When the distributions of $Y_1, \ldots, Y_{k_1}, \ldots, Y_K$ completely overlap, its easy to show that the chance curve is

$$Sen_o = (1 - (Spe_o(c))^{1/k_1})^{k_2}.$$

Figure 1 displays the ROC_o curve along with its corresponding chance curve. The marked symbols in Figure 1 will be discussed in Section 3.

2.2 | False classification rates

Both false positive rate and false negative rate under the current setting have not been formally defined, although Feng and Tian[26] proposed the concept of false negative rate under tree ordering, a special case of the current setting. In the following, we will present the definitions of false classification rates.

Define $U = \min(Y_1, \ldots, Y_{k_1})$, and $V = \max(Y_{k_1+1}, \ldots, Y_K)$. Then at any given $-\infty < c < \infty$, the overall false negative rate (FNR_o) can be defined as the probability that a randomly selected subject from any of the subclasses in the diseased class is falsely diagnosed into control class, and the overall false positive rate (FPR_o) as the probability that a randomly selected subject from any of the subclasses in the control class is falsely diagnosed into diseased class, that is

$$FNR_o(c) = P(V \leq c) = \prod_{j=k_1+1}^{K} F_{Y_j}(c), \tag{3}$$

$$FPR_o(c) = P(U > c) = \prod_{i=1}^{k_1} (1 - F_{Y_i}(c)). \tag{4}$$

Furthermore, we have

$$FNR_o(c) + Sen_o(c) = 1 - P(\min(Y_{k_1+1}, \ldots, Y_K) \leq c < \max(Y_{k_1+1}, \ldots, Y_K)), \tag{5}$$

$$FPR_o(c) + Spe_o(c) = 1 - P(\min(Y_1, \ldots, Y_{k_1}) \leq c < \max(Y_1, \ldots, Y_{k_1})). \tag{6}$$

Similar to the overall ROC (ROC_o) curve, we can generate a curve by plotting FPR_o against $(1 - FNR_o)$ for $-\infty < c < \infty$. We name this curve as "overall ROC false rates curve", which is displayed in Figure 2. The area under this curve equals to $P(\max(Y_{k_1+1}, \ldots, Y_K) \leq \min(Y_1, \ldots, Y_{k_1}))$, as proven in Appendix 1. The chance curve is $FPR_o = (1 - (FNR_o(c))^{1/k_2})^{k_1}$ which corresponds to the scenario that all the subclasses in both main classes overlap indicating the biomarker under consideration has no discriminatory accuracy.

3 | OPTIMAL THRESHOLD SELECTION METHODS WHEN SUBCLASSES ARE INVOLVED UNDER BINARY CLASSIFICATION

In this section, we propose three categories of optimal cut-point selection methods: A) methods constructed using correct classification rates, i.e. Sen_o and Spe_o; and B) methods constructed using false classification rates, i.e. FNR_o and FPR_o; C) a method which utilizes correction classification rates as well as false classification rates. The methods in category A are extensions of the optimal cut-point selection methods for tree ordering proposed by Wang et al.[23]; while the methods in categories B and C are original as they use newly defined false classification rates, FNR_o and FPR_o, in Section 2.2.

3.1 | Methods utilizing correct classification rates

3.1.1 | The Youden index (YI) method

The "overall Youden index" (YI_o) is defined as maximum of the sum of overall sensitivity (Sen_o) and overall specificity (Spe_o) minus 1, i.e.

$$YI_o = \max_c \left[Sen_o(c) + Spe_o(c) - 1\right] = \max_c \left[\prod_{j=k_1+1}^{K} (1 - F_{Y_j}(c)) + \prod_{i=1}^{k_1} F_{Y_i}(c) - 1\right], \tag{7}$$

where $Sen_O(c)$ and $Spe_O(c)$ are defined in (1) and (2), respectively. We denote the optimal cut-point determined by this method as c_{YI}. The maximum of YI_o is 1 corresponding to the scenario of perfect discrimination between two main classes, and the

minimum is 0 indicating the biomarker has no diagnostic ability. When $k_1 = k_2 = 1$, YI_o is the same as Youden index for traditional binary classification.

In Figure 1, the point $(1 - Spe_o(c_{YI}), Sen_o(c_{YI}))$ on "overall ROC" (ROC_o) curve corresponds to the optimal cut-point by the YI method, and YI_o is the length of the vertical line (color blue) from the point $(1 - Spe_O(c_{YI}), Sen_O(c_{YI}))$ to the diagonal line.

3.1.2 | The maximum area (MaxA) method

The maximum area under the overall ROC curve is defined as the maximum of rectangle area below the overall ROC curve with $Sen_o(c)$ as the length and $Spe_o(c)$ as the width, i.e.

$$MaxA = \max_c \left[Sen_o(c) \times Spe_o(c)\right] = \max_c \left[\left(\prod_{j=k_1+1}^{K} (1 - F_{Y_j}(c)) \right) \times \left(\prod_{i=1}^{k_1} F_{Y_i}(c) \right) \right]. \tag{8}$$

Let c_{MaxA} denote the optimal cut-point determined by the MaxA method. The largest value of $MaxA$ is one, indicating the scenario with perfect discrimination power. Note c_{MaxA} can also be determined by the following:

$$c_{MaxA} = \operatorname*{argmax}_c \left[\log(Sen_o(c)) + \log(Spe_o(c))\right].$$

In Figure 1, the point $(1 - Spe_o(c_{MaxA}), Sen_o(c_{MaxA}))$ on overall ROC curve corresponds to the optimal cut-point c_{MaxA} by the MaxA method, and $MaxA$ is the light purple shaded area.

Note that this method has a probabilistic interpretation, that is, it maximize the probability of a joint event in which both a randomly selected subject from control class and a randomly selected subject from disease class are correctly identified. In comparison with YI method, the MaxA method yields better balance between Sen_o and Spe_o.

3.1.3 | The northwest corner (NWC) method

The NWC method minimizes the distance from the perfection point $(0, 1)$ to $(1 - Spe_o(c), Sen_o(c))$ on the "overall ROC" curve in Figure 1. The shortest distance from the perfect point $(0, 1)$ to the overall ROC curve, denoted as D_{NWC}, is given as

$$\begin{aligned} D_{NWC} &= \min_c \left[\sqrt{(1 - Spe_o(c))^2 + (1 - Sen_o(c))^2} \right], \\ &= \min_c \left[\sqrt{(1 - \prod_{i=1}^{k_1} F_{Y_i}(c))^2 + (1 - \prod_{j=k_1+1}^{K} (1 - F_{Y_j}(c)))^2} \right]. \end{aligned} \tag{9}$$

Note D_{NWC} being 0 means perfect discrimination between two main classes, and increasing D_{NWC} means the biomarker accuracy gets weaker. We denote the optimal cut-point obtained by this method as c_{NWC}.

The NWC method has geometric appeal and can provide good balance between the correct classification rates for two main classes, i.e., $Sen_o(c)$ and $Spe_o(c)$. In Figure 1, the point with $(1 - Spe_o(c_{NWC}), Sen_o(c_{NWC}))$ on overall ROC curve corresponds to the optimal cut-point by the NWC method, and D_{NWC} is the length of the line (color red) from the point $(0, 1)$ to $(1 - Spe_o(c_{NWC}), Sen_o(c_{NWC}))$.

3.2 | Methods utilizing false classification rates

In binary classification, the false classification rates FNR and FPR covey same information as Sen and Spe, respectively, as $Sen(c)+FNR(c) = 1$ and $Spe(c)+FPR(c) = 1$. However, for the setting under consideration in this paper, i.e. binary classification with subclasses, from (5) and (6), we can see FNR_o and FPR_o convey information different from $Sen_o(c)$ and $Spe_o(c)$. Therefore, we can construct new optimal cut-point selection methods utilizing FNR_o and FPR_o.

3.2.1 | The overall minimum misclassification rate (MMR) method

The optimal cut-point can be determined by minimizing the sum of overall false negative rates across $-\infty < c < \infty$, i.e. the overall minimum misclassification rate (*MMR*) defined as

$$MMR = \min_c \left[FNR_o(c) + FPR_o(c)\right] = \min_c \left[\prod_{j=k_1+1}^{K} F_{Y_j}(c) + \prod_{i=1}^{k_1}(1 - F_{Y_i}(c))\right]. \tag{10}$$

where $FNR_o(c)$ and $FPR_o(c)$ are defined in (3) and (4), respectively. We denote c_{MMR} as the optimal cut-point determined by this method. In Figure 2, the point $(1 - FNR_o(c_{MMR}), FPR_o(c_{MMR}))$ on "overall ROC false rates curve" corresponds to the optimal cut-point selected by the MMR method, and *MMR* equals to one minus the length of the vertical line (color blue) from the point $(1 - FNR_o(c_{MMR}), FPR_o(c_{MMR}))$ to the diagonal line.

3.2.2 | The minimum area (MinA) method

The minimum area under the "overall ROC false rates curve" is defined as the minimum of the product of FNR_o and FPR_o, i.e., the minimum rectangle area under the curve (denoted as *MinA*) with FNR_o as the length and FPR_o as the width. The optimal cut-point selected by this method, which is denoted as c_{MinA}, can be obtained by the following criterion:

$$MinA = \min_c \left[FNR_o(c) \times FPR_o(c)\right] = \min_c [\prod_{j=k_1+1}^{K} F_{Y_j}(c) \times \prod_{i=1}^{k_1}(1 - F_{Y_i}(c))]. \tag{11}$$

The MinA method determines a cut-point c_{MinA} which yields the minimum of the probability of a joint event in which both a randomly selected subject from any subclasses in control class and a randomly selected subject from any subclasses in diseased class are falsely classified. Note c_{MinA} can also be obtained by the following:

$$c_{MinA} = \operatorname*{argmin}_c \left[\log(FNR_o(c)) + \log(FPR_o(c))\right].$$

In Figure 2, the point $(1 - FNR_o(c_{MinA}), FPR_o(c_{MinA}))$ on "overall ROC false rates curve" corresponds to the optimal cut-point selected by the MinA method, and *MinA* is the purple shaded area.

3.2.3 | The southeast corner (SEC) method

The SEC method selects the cut-point c_{SEC} such that the point $(1 - FNR_o(c_{SEC}), FPR_o(c_{SEC}))$ on the "overall ROC false rates curve" is closest to the southeast corner point $(1, 0)$, i.e., where both FNR_o and FPR_o equal to 0. The minimum distance distance D_{SEC} from $(1, 0)$ to $(1 - FNR_o(c), FPR_o(c))$ can be written as

$$\begin{aligned} D_{SEC} &= \min_c \left[\sqrt{FNR_o(c)^2 + FPR_o(c)^2}\right] \\ &= \min_c \left[\sqrt{(\prod_{j=k_1+1}^{K} F_{Y_j}(c))^2 + (\prod_{i=1}^{k_1}(1 - F_{Y_i}(c)))^2}\right], \end{aligned} \tag{12}$$

representing the shortest distance from the perfect point $(1, 0)$ to the "overall ROC false rates curve" . Note this perfection point is defined differently from the perfection point on overall ROC curve. The D_{SEC} with value $\sqrt{2}$ indicates no discrimination while decreasing D_{SEC} indicates better ability of discriminating between two main classes.

In Figure 2, the point with coordinates $(1 - FNR_o(c_{SEC}), FPR_o(c_{SEC}))$ on "overall ROC false rates curve" corresponds to the optimal cut-point by the SEC method, and D_{SEC} is the length of the line (color red) from the point $(1, 0)$ to $(1 - FNR_o(c_{SEC}), FPR_o(c_{SEC}))$.

3.3 | The penalized overall Youden index (PYI) method

A desirable cut-point selection method can be developed by making use of both correct classification rates (i.e., Sen_o and Spe_o) and false classification rates (i.e., FNR_o and FPR_o). Hence we propose a new cut-point selection method, using the objective

function defined as

$$PYI = \max_c \left[Sen_o(c) + Spe_o(c) - 1 - (FNR_o(c) + FPR_o(c))\right],$$
$$= \max_c \left[\prod_{j=k_1+1}^{K}(1 - F_{Y_j}(c)) + \prod_{i=1}^{k_1} F_{Y_i}(c) - 1 - (\prod_{j=k_1+1}^{K} F_{Y_j}(c) + \prod_{i=1}^{k_1}(1 - F_{Y_i}(c)))\right]. \quad (13)$$

Note that PYI can be viewed as overall Youden index penalized by the false classification rates.

Let c_{PYI} denote the optimal cut-point determined by the PYI method. Similar idea has been used in multiclass classification under simple ordering by Hua and Tian[18].

4 | CONFIDENCE INTERVAL ESTIMATION FOR OPTIMAL CUT-POINTS

In this section, we study confidence interval estimation for optimal cut-points generated by the methods proposed in Section 3. Parametric methods are presented in Section 4.1 and 4.2, and nonparametric methods are presented in Section 4.3.

4.1 | Under normality

In this section, we will propose generalized confidence intervals under normality for the optimal cut-points using the concept of generalized pivotal quantity for the normal data. Tsui and Weerahandi[27] and Weerahandi[28] introduced the concepts of generalized variables and generalized pivots. The generalized inference methods generally have good performance even at small sample size, hence have been widely applied to many different problems where exact solutions do not exist.[29,30,31] More details on generalized inference can be found in the book by Weerahandi.[32]

Let $Y_1, \ldots, Y_{k_1}$ stand for the random variable for biomarker values for control subclasses $1, \ldots, k_1$, respectively, and $Y_{k_1+1}, \ldots, Y_K$ for diseased subclasses $k_1 + 1, \ldots, K$, respectively. Assume $Y_k \sim N(\mu_k, \sigma_k^2)$, where $k = 1, \ldots, k_1, k_1 + 1, \ldots, K$. Using the definitions in (1), (2), (3), and (4) presented in Section 2, under normality assumptions, the overall sensitivity (Sen_o), overall specificity (Spe_o), overall false negative rate ($FNR_o(c)$), and overall false positive rate ($FPR_o(c)$) can be written as follows:

$$Sen_o(c) = \prod_{j=k_1+1}^{K}(1 - \Phi(\frac{c - \mu_j}{\sigma_j})), \quad Spe_o(c) = \prod_{i=1}^{k_1}\Phi(\frac{c - \mu_i}{\sigma_i}),$$
$$FNR_o(c) = \prod_{j=k_1+1}^{K}\Phi(\frac{c - \mu_j}{\sigma_j}), \quad FPR_o(c) = \prod_{i=1}^{k_1}(1 - \Phi(\frac{c - \mu_i}{\sigma_i})).$$

Furthermore, under normality, the objective functions YI_o, $MaxA$, D_{NWC}, MMR, $MinA$, D_{SEC}, and PYI proposed in Section 3 can be expressed as follows:

$$
\begin{aligned}
YI_o &= \max_c \left[\prod_{j=k_1+1}^{K} (1-\Phi(\frac{c-\mu_j}{\sigma_j})) + \prod_{i=1}^{k_1} \Phi(\frac{c-\mu_i}{\sigma_i}) - 1 \right], \\
MaxA &= \max_c \left[\sum_{j=k_1+1}^{K} \log(1-\Phi(\frac{c-\mu_j}{\sigma_j})) + \sum_{i=1}^{k_1} \log(\Phi(\frac{c-\mu_i}{\sigma_i})) \right], \\
D_{NWC} &= \min_c \left[\sqrt{(1-\prod_{i=1}^{k_1} \Phi(\frac{c-\mu_i}{\sigma_i}))^2 + (1-\prod_{j=k_1+1}^{K} (1-\Phi(\frac{c-\mu_j}{\sigma_j})))^2} \right], \\
MMR &= \min_c \left[\prod_{j=k_1+1}^{K} \Phi(\frac{c-\mu_j}{\sigma_j}) + \prod_{i=1}^{k_1} (1-\Phi(\frac{c-\mu_i}{\sigma_i})) \right], \\
MinA &= \min_c \left[\sum_{j=k_1+1}^{K} \log(\Phi(\frac{c-\mu_j}{\sigma_j})) + \sum_{i=1}^{k_1} \log(1-\Phi(\frac{c-\mu_i}{\sigma_i})) \right] \\
D_{SEC} &= \min_c \left[\sqrt{(\prod_{j=k_1+1}^{K} \Phi(\frac{c-\mu_j}{\sigma_j}))^2 + (\prod_{i=1}^{k_1} (1-\Phi(\frac{c-\mu_i}{\sigma_i})))^2} \right], \\
PYI &= \max_c \left[\prod_{j=k_1+1}^{K} (1-\Phi(\frac{c-\mu_j}{\sigma_j})) + \prod_{i=1}^{k_1} \Phi(\frac{c-\mu_i}{\sigma_i}) - (\prod_{j=k_1+1}^{K} \Phi(\frac{c-\mu_j}{\sigma_j}) + \prod_{i=1}^{k_1} (1-\Phi(\frac{c-\mu_i}{\sigma_i}))) \right].
\end{aligned}
$$

Table 1 lists optimal cut-points obtained by proposed methods under normality.

Let $\bar{Y}_k$ be the sample mean and S_k^2 be the sample variance, and $\bar{y}_k$ and s_k^2 be the corresponding observed values. The generalized pivotal quantities for normal variances and mean are well known as[29]

$$
R_{\sigma_k^2} = \frac{(n-1)s_k^2}{V_k}, R_{\mu_k} = \bar{y}_k - Z_k \sqrt{R_{\sigma_k^2}/n_k},
$$

where $V_k = \frac{(n_k-1)S_k^2}{\sigma_k^2} \sim \chi^2_{n_k-1}$ and $Z_k = \frac{\sqrt{n_k}(\bar{Y}_k-\mu_k)}{\sigma_k} \sim N(0,1)$, for $k = 1, \ldots, K$. The generalized pivotal quantity for σ_k is defined as $R_{\sigma_k} = \sqrt{R_{\sigma_k^2}}$. Consequently, the generalized pivotal quantity R_c for a optimal cut-point c in Table 1 can be obtained by substituting μ_k and σ_k with corresponding generalized pivotal quantities. Given a specific data set $Y_{k,h}$ ($k = 1, 2, \ldots, K$, and $h = 1, \ldots, n_k$), the confidence region for R_c by generalized inference approach can be obtained via the following steps : (1) Compute the sample mean $\bar{y}_k$ and sample variance s_k^2, for $k = 1, \ldots, K$; (2) Generate V_k from $\chi^2_{n_k-1}, k = 1, \ldots, K$; (3) Compute R_{μ_k} and R_{σ_k};(4) Calculate R_c for different methods listed in table 1; (5) Repeat steps (2) - (4) for $B = 2500$ times to obtain a set of values of R_c^b for $b = 1, 2, \ldots, B$; (6) Compute the $100(\alpha/2)^{th}$ percentile $R_{c,\alpha/2}$ and $100(1-\alpha/2)^{th}$ percentile $R_{c,1-\alpha/2}$ of $R_{c_1}, \ldots, R_{c_B}$. then $(R_{c,\alpha/2}, R_{c,1-\alpha/2})$ is the $100(1-\alpha)\%$ confidence interval of c. We denote such confidence interval estimation method as generalized inference (**GI**) method.

4.2 | Under gamma distribution

In practice, we encounter scenarios that biomarker measurements are continuous and positively skewed. Gamma distribution is well known to be a popular option for modeling rignt-skewed data. Therefore, we present some generalized inference methods for constructing the confidence intervals of optimal cut-points by proposed methods under gamma distribution.

Let $Y_k \sim G(\alpha_k, \beta_k)$, where $\alpha_k > 0$ (shape parameter), and $\beta_k > 0$ (rate parameter). Overall sensitivity (Sen_o), overall specificity (Spe_o), overall false negative rate (FNR_o), and overall false positive rate (FPR_o) are defined by utilizing gamma c.d.f., $F_k(x \mid \alpha_k, \beta_k) = \gamma(\alpha_k, \beta_k x)/\Gamma(\alpha_k)$, where $\Gamma(\alpha_k)$ stands for gamma function $\int_0^\infty t^{\alpha_k-1}e^{-t}dt$, and $\gamma(\alpha_k, \beta_k x)$ stands for lower gamma

function $\int_0^{\beta_k x} t^{\alpha_k-1}e^{-t}dt$. Specifically,

$$Sen_o(c) = \prod_{j=k_1+1}^{K} \frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)} \int_c^{\infty} u^{\alpha_j-1}e^{-\beta_j u}du, \quad Spe_o(c) = \prod_{i=1}^{k_1} \frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)} \int_0^{c} u^{\alpha_i-1}e^{-\beta_i u}du$$

$$FNR_o(c) = \prod_{j=k_1+1}^{K} \frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)} \int_0^{c} u^{\alpha_j-1}e^{-\beta_j u}du, \quad FPR_o(c) = \prod_{i=1}^{k_1} \frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)} \int_c^{\infty} u^{\alpha_i-1}e^{-\beta_i u}du$$

Furthermore, YI_o, $MaxA$, D_{NWC}, MMR, $MinA$, D_{SEC}, and PYI can be expressed by using above functions in (7), (8), (9), (10), (11), (12), and (13), respectively. Table 2 presents the optimal cut-points obtained by different selection method under gamma distribution.

Let $Y_1, \ldots, Y_n$ be an iid random sample from $G(\alpha, \beta)$. Let $\bar{Y}$ and $\tilde{Y}$ stand for the arithmetic mean and geometric mean, and $\bar{y}$ and $\tilde{y}$ be the observed values of $\bar{Y}$ and $\tilde{Y}$, respectively. While the exact fiducial quantity for shape parameter (α) and rate parameter (β) in gamma distribution is unavailable, several versions of approximate pivotal quantities (GPQ) for α and β were presented.[33,34,35,36] We will review them briefly in the following.

Chen and Ye's method[33]: It is known that $2n\alpha \log(\bar{Y}/\tilde{Y}) \sim \alpha\chi^2_{\upsilon}$ approximately, where $\upsilon = 2E^2(W)/\text{var}(W)$ and $c = E(W/\upsilon)$. The detailed formulas for $E(W)$ and $\text{var}(W)$ can be found in Chen and Ye.[33] Using this result, an approximate generalized pivotal quantity for α can be written as

$$R_\alpha = \frac{W}{2n\log(\bar{y}/\tilde{y})},$$

where $W \sim c\chi^2_{(\upsilon)}$. Furthermore, utilizing a well-known result regarding gamma distribution, i.e. $2n\beta\bar{Y} \sim \chi^2_{2n\alpha}$, the generalized pivot quantity for β_k can be written as

$$R_\beta = \frac{U}{2n\bar{y}}, \tag{14}$$

where $U \sim \chi^2_{2nR_\alpha}$.

Wang and Wu's method[34]: Let $T = \log(\tilde{Y}/\bar{Y})$. Note that $U = F(.) \sim U(0,1)$, where $F(.)$ is the c.d.f of T. On the basis of Cornish-Fisher expansion, the Uth percentile of T can be approximated by $\kappa_1(\alpha) + [\kappa_2(\alpha)]^{1/2}Q(\alpha, U)$, where $\kappa_j(\alpha)$ is the jth cumulant of T and $Q(\alpha, U)$ is a function of $\kappa_j(\alpha)$. The detailed formulas can be found in Wang and Wu[34]. Let t denote the observed value of T. An approximate generalized pivotal quantity for α, i.e. R_α, can be obtained by solving $t = \kappa_1(\alpha) + [\kappa_2(\alpha)]^{1/2}Q(\alpha, U)$. Similar to Chen and Ye's method, the approximate generalized pivotal quantity for rate parameter, R_β, can be obtained by (14).

Krishnamoorthy and Wang's method[35,36]: By applying the Wilson-Hilferty normal approximation, i.e. $Y^{1/3} \sim N(\mu, \sigma)$. Generalized pivotal quantities for normal mean and variance, R_μ and R_σ can be obtained for transformed data. The GPQs for α and β can be further expressed as:

$$R_\alpha = \frac{1}{9}\{(1 + 0.5\frac{R_\mu^2}{R_\sigma^2}) + [(1 + 0.5\frac{R_\mu^2}{R_\sigma^2})^2 - 1]^{1/2}\},$$

$$R_\beta = \frac{1}{27(R_\alpha)^{1/2}(R_{\sigma^2})^{3/2}}.$$

For each class under gamma distribution, the generalized pivots of optimal cut-points by proposed methods can be obtained by substituting α_k and β_k in Table 2 with corresponding R_{α_k} and R_{β_k} $(k = 1, 2, \ldots, K)$ by these approximate generalized inference methods presented above. Following similar steps as presented in Section 4.1 for normality, we can obtain the confidence intervals for proposed optimal cut-points under gamma distribution. We refer these three generalized inference methods for gamma distribution as: **GammaGI1** (based on Chen and Ye's method), **GammaGI2** (based on Wang and Wu's method), and **GammaGI3** (based on Krishnamoorthy and Wang's method).

4.3 | Non-parametric confidence interval estimation

The parametric methods presented in 4.1 and 4.2 need parametric assumptions of data which are not always feasible. Therefore, it is important to evaluate the performance of non-parametric approach for estimating confidence interval of optimal cut-points.

Given an observed data set $Y_{k,h}$'s where $k = 1, 2, \ldots, K$, and $h = 1, 2, \ldots, n_K$, the empirical estimate of overall sensitivity ($\widehat{Sen}_o$), overall Specificity ($\widehat{Spe}_o$), overall false negative rate ($\widehat{FNR}_o$), and overall false positive rate ($\widehat{FPF}_o$) can be written as

$$\widehat{Sen}_o = \prod_{j=k_1+1}^{K}(1 - \frac{1}{n_j}\sum_{m=1}^{n_j} I(Y_{j,m} \le c)), \quad \widehat{Spe}_o = \prod_{i=1}^{k_1}(\frac{1}{n_i}\sum_{l=1}^{n_i} I(Y_{i,l} \le c)),$$

$$\widehat{FNR}_o = \prod_{j=k_1+1}^{K}(\frac{1}{n_j}\sum_{m=1}^{n_j} I(Y_{j,m} \le c)), \quad \widehat{FPR}_o = \prod_{i=1}^{k_1}(1 - \frac{1}{n_i}\sum_{l=1}^{n_i} I(Y_{i,l} \le c)).$$

Furthermore, the empirical estimate of YI_o, $MaxA$, D_{NWC}, MMR, $MinA$, D_{SEC}, and PYI defined in (7), (8), (9), (10), (11), (12), and (13) can be expressed using $\widehat{Sen}_o$, $\widehat{Spe}_o$, $\widehat{FNR}_o$, and $\widehat{FPR}_o$. For example, the empirical estimate of YI_o can be written as

$$\widehat{YI}_o(c) = \prod_{j=k_1+1}^{K}(1 - \frac{1}{n_j}\sum_{m=1}^{n_j} I(Y_{j,m} \le c)) + \prod_{i=1}^{k_1}(\frac{1}{n_i}\sum_{l=1}^{n_i} I(Y_{i,l} \le c)), \tag{15}$$

The $100(1-\alpha)\%$ bootstrap confidence interval for the optimal cut-point c can be obtained as follows: (1) For each class k, resample with replacement n_k observations from $y_{k,1}, y_{k,2}, \ldots, y_{k,n_k}$, and denote the bootstrap samples as $\{Y_{k,h}\}, h = 1, 2, \ldots, n_k, k = 1, 2, \ldots, K$; (2) Derive estimate on c from each set of bootstrap samples by minimizing or maximizing different objective functions; (3) Repeat steps 1-2 a total of $B = 2000$ times and obtain a set of $\hat{c}$; (4) Rank the array $\hat{c}$ from small to large. A two-sided $100(1-\alpha)\%$ confidence interval estimate of c is $(\hat{c}(\alpha/2), \hat{c}(1-\alpha/2))$. This approach is referred as non-parametric bootstrap (**NPB**) method.

5 | SIMULATION STUDIES

The simulation studies contains two parts. In 5.1, we evaluate the performances of the optimal cut-points selection methods proposed in Section 3. In 5.2, we evaluate the coverage probabilities and the mean lengths of the confidence interval estimation methods proposed in Section 4. Table 3 presents parameter settings under normality and gamma distribution. Figure 3 and Figure 4 present the corresponding density plots for normality and gamma distribution, respectively.

5.1 | Performance of selection methods

In this section, we present the simulation results to evaluate the performance of the proposed methods for selecting the optimal cut-point when subclasses are involved under binary classification. For each parameter setting, a total of $R = 10,000$ iterations were used. Let c_i stand for the estimate at i th iteration, c the true optimal cut-point, and $\hat{c} = \frac{1}{R}\sum_{i=1}^{R} c_i$ (i.e. the estimated optimal cut-point). The following indexes are used to evaluate the performance of all proposed methods:

1. Relative Bias (*RB*)
$$RB(\hat{c}) = \frac{1}{R}\sum_{i=1}^{R}\frac{c_i - c}{c}.$$
2. Root Mean Square Error (*RMSE*)
$$RMSE(\hat{c}) = \sqrt{Bias^2(\hat{c}) + \mathrm{Var}(\hat{c})},$$
where $Bias(\hat{c}) = \frac{1}{R}\sum_{i=1}^{R} c_i - c$ and $\mathrm{Var}(\hat{c}) = \sum_{i=1}^{R}(c_i - c)^2/(R-1)$.
3. Total Correct Classification Rate (*TCCR*)
$$TCCR = \frac{1}{R}\sum_{i=1}^{R} TCCR_i, \tag{16}$$
where $TCCR_i = Spe_o(c_i) + Sen_o(c_i)$.
4. Maximum Minimum Relative Difference (*MMRDIF*) : *MMRIDF* measures the degree of imbalance between Sen_o and Spe_o at optimal cut-point.
$$MMRDIF = \frac{1}{R}\sum_{i=1}^{R} MMRDIF_i, \tag{17}$$

where $MMRDIF_i = [\max(Sen_o(c_i), Spe_o(c_i)) - \min(Sen_O(c_i), Spe_o(c_i))]/\min(Sen_o(c_i), Spe_o(c_i))$. A smaller *MMRDIF* means better balance level between overall sensitivity and overall specificity.

Tables 4 and 5 present the simulation results of the relative bias (*RB*) and root mean square error (*RMSE*) for the optimal cut-point estimates under normal distributions, respectively. As sample sizes go up, both bias and *RMSE* decrease. The MinA method generally performs the best as it has the smallest *RB* for most scenarios and the smallest *RMSE* for all scenarios.

Tables 6 and 7 present the simulation results of the relative bias (*RB*) and root mean square error (*RMSE*) for the optimal cut-point estimates under gamma distributions, respectively. As sample sizes increase, both bias and *RMSE* decrease. The MinA method has the smallest *RMSE* for most scenarios. The YI method performs worst in terms of *RB* for all scenarios, and there is no clear winner among all other methods across different scenarios.

Tables 8 and 9 present the estimates of total correct classification rate (*TCCR*) and maximum minimum relative difference (*MMRDIF*) under normal distributions, respectively. The YI method has the largest *TCCR* among all methods as *TCCR* is its objective function. The PYI method generally has better *TCCR* than the rest of methods. The MinA method has the best performance in terms of balance between Sen_o and Spe_o for all scenarios.

Tables 10 and 11 present the estimates of total correct classification rate (*TCCR*) and maximum minimum relative difference (*MMRDIF*) under gamma distributions, respectively. Similarly, the YI method has the largest *TCCR* in all scenarios followed by the PYI method. The MinA method has the best performance with respect to balance between Sen_o and Spe_o for most scenarios, while the NWC method performs the best in balancing Sen_o and Spe_o for when AUC_o is larger than 0.07.

Overall, the MinA method has good performance regarding *RMSE* and *MMRDIF* as well as the balance between Sen_o and Spe_o. In terms of *TCCR*, the YI is by defaut the best followed by the PYI method.

5.2 | Confidence intervals

Simulation studies are performed to evaluate the coverage probabilities and the average lengths of the confidence intervals of optimal cut-points by proposed methods. For each parameter setting, 2000 samples are simulated. For generalized confidence intervals, we set $B = 2500$. For bootstrap methods, $B = 500$ bootstrap samples are used.

Table 12 presents estimated coverage probabilities at the nominal level of 95% and the estimated average lengths of the confidence intervals for optimal cut-points under normality from small to large sample sizes. The **GI** method generally achieves satisfactory coverage probabilities for confidence intervals of the optimal cut-points obtained by all proposed methods. But the coverage probabilities of the confidence intervals for c_{YI} (optimal cut-point obtained by YI method) could be slightly conservative when sample sizes are smaller than $(30, 30, 30, 30)$. The confidence intervals by **NPB** method are generally liberal when sample sizes are smaller than $(50, 50, 50, 50)$. In terms of estimated average lengths of confidence intervals, the generalized inference (**GI**) method yield smaller lengths than non-parametric bootstrap (**NPB**) method.

Table 13 presents estimated coverage probabilities at the nominal level of 95% and the estimated average lengths of the confidence intervals for optimal cut-points under gamma distributions from small to large sample sizes. The three approximate generalized inference methods (**GammaGI1**, **GammaGI2**, and **GammaGI3**) maintain satisfied coverage probabilities under most scenarios for confidence intervals of the optimal cut-points obtained by proposed method, except the YI method. The coverage probabilities for c_{YI} is very conservative when sample sizes are smaller than $(50, 50, 50, 50)$, especially for small AUC_o. The **NPB** method is generally liberal for all cut-point selection methods when sample sizes are smaller than $(30, 30, 30, 30)$, especially for the c_{YI}. As sample sizes increase, the **NPB** coverage probabilities of confidence interval of cut-points selected by proposed methods tend to be satisfied. In respect of lengths of confidence intervals, the three approximate generalized inference methods (**GammaGI1**, **GammaGI2**, and **GammaGI3**) achieve smaller average length of confidence intervals in comparing with non-parametric bootstrap (**NPB**) method for all proposed cut-point selection methods.

Overall, **GI** has more accurate coverage probabilities and smaller estimated average length than **NPB** under normality for proposed optimal cut-point selection methods. Under gamma distributions, **GammaGI1**, **GammaGI2**, and **GammaGI3** outperform **NPB** for proposed optimal cut-point selection methods except the YI method. When using the YI method to determine the optimal cut-point under gamma distributions, **NPB** is a good choice.

6 | EXAMPLE

Ovarian cancer is the eighth most common and the fifth leading cause of cancer death in women worldwide.[37] The Prostate, Lung, Colorectal and Ovarian (PLCO) Cancer Study is a randomized trial to determine the effect on cancer mortality by evaluating screening modalities.[25] Women who were assigned to the ovarian screening arm received annual transvaginal ultrasound (TVU) and CA125 testing. A phase II study under the Early Detection Research Network (EDRN) initiated by four sites in ovarian cancer included 80 late-staged cases (CaseLate), 80 early-staged cases (CaseEarly), 160 controls with benign disease (BDcontrol), and 480 general population controls (GPcontrol). The data set can be accessed from public portal at https://edrn.nci.nih.gov, and the results have been published in many research papers.[25,38] Cramer et al.[25] evaluated a number of biomarker candidates using binary ROC analysis by pooling all general healthy class (GPcontrol) and benign cases (BDcontrol) to create a non-cancer class, and all early-staged cases (CaseEarly) and late-staged cases (CaseLate) to create a cancer class. Feng and Tian[24] re-evaluated the biomarker candidates by using overall AUC (AUC_o), a proper measure for assessing diagnostic accuracy to classify two main classes.

We use CA125 and IGF2 to illustrate the proposed optimal cut-point selection methods for distinguishing non-cancer cases (BDcontrol and GPcontrol) and cancer cases (CaseLate and CaseEarly). While CA125 is a well known biomarkers for ovarian cancer ($AUC_o = 0.705$), IGF2 has weaker diagnostic accuracy ($AUC_o = 0.458$).[24] For both CA125 and IGF2, neither normality nor gamma assumption are satisfied based on Shapiro-Wilk test gamma goodness-of-fit test.[39] Thus, non-parametric bootstrap (**NPB**) method is used to estimate the confidence intervals of optimal cut-points.

Table 14 and Table 15 present estimated optimal cut-points by proposed methods along with corresponding 95% **NPB** confidence intervals, sensitivity (Sen_o), specificity (Spe_o), and total correct classification rate (CCR_o) at optimal cut-point, for CA125 and IGF2, respectively. Figure 5 and Figure 6 present density plots by subclasses along with estimated optimal cut-points selected by the proposed methods, for CA125 and IGF2, respectively. For CA125, all the cut-point selection methods generally agree with each other, and yield good balance between Sen_o and Spe_o, possibly due to the decent sample sizes and the good diagnostic performance of CA125. However, for IGF2, the YI method generates highly unbalanced correct classification rates with Sen_obeing 0.001 and Spe_o being 1.000, while the rest of the methods perform similarly with good balance between Sen_o and Spe_o.

7 | SUMMARY AND DISCUSSION

In cancer diagnostic studies, one common setting is binary classification (e.g. non-cancer v.s. cancer) where both main classes consist multiple subclasses. While research has been done for biomarkers evaluation for their classification accuracy between two main classes,[24] the optimal cut-point selection methods for such setting have never been addressed.

This paper addresses the issue of cut-point selection methods under such setting. A large number os methods are proposed, based on both either existing measures and newly defined measures. Additionally, parametric and nonparametric inference methods are studied for estimating confidence intervals of the proposed cut-point selection methods. While YI method and PYI method are the most accurate methods in terms of overall accuracy, the resulting Sen_O and Spe_o can be severely unbalanced. Overall, via a simulation, we observe that the newly proposed minimum area (MinA) method is the most noteworthy method considering all the performance criteria. The Youden index (YI) method, despite its popularity in practice and research, can generate highly unbalanced correct classification rates. The penalized Younden index (PYI) index method is another reasonable option.

While the cut-point selection methods proposed in this paper are illustrated by a single biomarker in the example, certainly the proposed cut-point selection methods can apply to a combined biomarker which can be derived from a variety biomarker combination methods. However, the confidence interval estimation methods for the optimal cut-points for a combined marker could be quite challenging taking in to account of the variability from estimated combination coefficients.

An R-program is available at request from Dr. Tian at ltian@buffao.edu.

ACKNOWLEDGMENTS

Data used in preparation of this article were obtained from the Early Detection Research Network (EDRN) database (http://www.cancer.gov/edrn).

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APPENDIX A. Proof of area under the "Overall ROC false rate curve" $(1 - FNR_o, FPR_o)$ proposed in Section 2.2)

Let AUC_{OF} denote the area under the "Overall ROC false rate curve" proposed in Section 2.2. We have

$$
\begin{aligned}
AUC_{OF} &= \int_0^1 FPR_o d(1 - FNR_o) \\
&= \int_{-\infty}^{\infty} \prod_{i=1}^{k_1} (1 - F_i(c) d(1 - \prod_{j=k_1+1}^{K} F_j(c)) \\
&= \sum_{p=k_1+1}^{K} \int_{-\infty}^{\infty} \prod_{i=1}^{k_1} (1 - F_i(c) \prod_{j=k_1+1, j\neq p}^{K} F_j(c) f_j(c) dc \\
&= \sum_{p=k_1+1}^{K} E_{Y_p} \left[\prod_{i=1}^{k_1} P(Y_i > c \mid Y_p = c) \prod_{j=k_1+1, j\neq p}^{K} P(Y_j \leq c \mid Y_p = c) \right] \\
&= \sum_{p=k_1+1}^{K} E_{Y_p} \left[P(V > c \mid Y_p = c) P(U \leq c \mid Y_p = c) \right] \\
&= P(U \leq V) \\
&= P(max(Y_{k_1+1}, \ldots, Y_K) \leq min(Y_1, \ldots, Y_{k_1}))
\end{aligned}
$$

TABLES

Table 1. Optimal cut-points by different selection methods under normality.

Method	Optimal cut-point c
YI	$\underset{c}{\text{argmax}}[\prod_{j=k_1+1}^{K}(1-\Phi(\frac{c-\mu_j}{R_{\sigma_j}}))+\prod_{i=1}^{k_1}\Phi(\frac{c-\mu_i}{\sigma_i})]$
MaxA	$\underset{c}{\text{argmax}}[\sum_{j=k_1+1}^{K}\log(1-\Phi(\frac{c-\mu_j}{\sigma_j}))+\sum_{i=1}^{k_1}\log(\Phi(\frac{c-\mu_i}{\sigma_j}))]$
NWC	$\underset{c}{\text{argmin}}[(1-\prod_{j=k_1+1}^{K}(1-\Phi(\frac{c-\mu_j}{\sigma_j})))^2+(1-\prod_{i=1}^{k_1}\Phi(\frac{c-\mu_i}{\sigma_i}))^2]$
MMR	$\underset{c}{\text{argmin}}[\prod_{j=k_1+1}^{K}\Phi(\frac{c-\mu_j}{\sigma_j})+\prod_{i=1}^{k_1}(1-\Phi(\frac{c-\mu_i}{\sigma_i}))]$
MinA	$\underset{c}{\text{argmin}}[\sum_{j=k_1+1}^{K}\log(\Phi(\frac{c-\mu_j}{\sigma_j}))+\sum_{i=1}^{k_1}\log(1-\Phi(\frac{c-\mu_i}{\sigma_i}))]$
SEC	$\underset{c}{\text{argmin}}[(\prod_{j=k_1+1}^{K}\Phi(\frac{c-\mu_j}{\sigma_j}))^2+(\prod_{i=1}^{k_1}(1-\Phi(\frac{c-\mu_i}{\sigma_i})))^2]$
PYI	$\underset{c}{\text{argmax}}[\prod_{k_1+1}^{K}(1-\Phi(\frac{c-\mu_j}{\sigma_j}))-\prod_{j=k_1+1}^{K}\Phi(\frac{c-\mu_j}{\sigma_j})+\prod_{i=1}^{k_1}\Phi(\frac{c-\mu_i}{\sigma_i})-\prod_{i=1}^{k_1}(1-\Phi(\frac{c-\mu_i}{\sigma_i}))]$

Table 2. Optimal cut-points by different selection methods under gamma distribution.

Method	Optimal cut-point c
YI	$\underset{R_c}{\text{argmax}}[\prod_{j=k_1+1}^{K}\frac{R_{\beta_j}^{R_{\alpha_j}}}{\Gamma(R_{\alpha_j})}\int_{R_c}^{\infty}u^{R_{\alpha_j}-1}e^{-R_{\beta_j}u}du+\prod_{i=1}^{k_1}\frac{R_{\beta_i}^{R_{\alpha_i}}}{\Gamma(R_{\alpha_i})}\int_0^c u^{R_{\alpha_i}-1}e^{-R_{\beta_i}u}du-1]$
MaxA	$\underset{c}{\text{argmax}}[\sum_{j=k_1+1}^{K}\log(\frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)}\int_c^{\infty}u^{\alpha_j-1}e^{-\beta_j u}du)+\sum_{i=1}^{k_1}\log(\frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)}\int_0^c u^{\alpha_i-1}e^{-\beta_i u}du)]$
NWC	$\underset{c}{\text{argmin}}[(1-\prod_{j=k_1+1}^{K}(1-\prod_{j=k_1+1}^{K}\frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)}\int_c^{\infty}u^{\alpha_j-1}e^{-\beta_j u}du)^2+(1-\prod_{i=1}^{k_1}\frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)}\int_0^c u^{\alpha_i-1}e^{-\beta_i u}du)^2]$
MMR	$\underset{c}{\text{argmin}}[\prod_{j=k_1+1}^{K}\frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)}\int_0^c u^{\alpha_j-1}e^{-\beta_j u}du+\prod_{i=1}^{k_1}\frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)}\int_c^{\infty}u^{\alpha_i-1}e^{-\beta_i u}du]$
MinA	$\underset{c}{\text{argmin}}[\sum_{j=k_1+1}^{K}\log(\frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)}\int_0^c u^{\alpha_j-1}e^{-\beta_j u}du)+\sum_{i=1}^{k_1}\log(\frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)}\int_c^{\infty}u^{\alpha_i-1}e^{-\beta_i u}du)]$
SEC	$\underset{c}{\text{argmin}}[(\prod_{j=k_1+1}^{K}\frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)}\int_0^c u^{\alpha_j-1}e^{-\beta_j u}du)^2+(\prod_{i=1}^{k_1}\frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)}\int_c^{\infty}u^{\alpha_i-1}e^{-\beta_i u}du)^2]$
PYI	$\underset{c}{\text{argmax}}[\prod_{j=k_1+1}^{K}\frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)}\int_c^{\infty}u^{\alpha_j-1}e^{-\beta_j u}du+\prod_{i=1}^{k_1}\frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)}\int_0^c u^{\alpha_i-1}e^{-\beta_i u}du-$ $\prod_{j=k_1+1}^{K}\frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)}\int_0^c u^{\alpha_j-1}e^{-\beta_j u}du-\prod_{i=1}^{k_1}\frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)}\int_c^{\infty}u^{\alpha_i-1}e^{-\beta_i u}du]$

Table 3. Simulation settings ($k_1 = k_2 = 2$).

Scenario	Parameter settings				
	Class 1	Class 2	Class 3	Class 4	AUC_o
Normal 1	$N(1.00, 1.00^2)$	$N(1.00, 1.20^2)$	$N(1.50, 1.20^2)$	$N(1.50, 1.40^2)$	0.275
Normal 2	$N(1.00, 1.00^2)$	$N(1.00, 1.00^2)$	$N(1.50, 1.00^2)$	$N(1.50, 1.00^2)$	0.299
Normal 3	$N(0.00, 1.00^2)$	$N(1.00, 1.00^2)$	$N(1.50, 1.00^2)$	$N(2.00, 1.20^2)$	0.474
Normal 4	$N(0.50, 1.00^2)$	$N(0.75, 1.00^2)$	$N(1.75, 1.20^2)$	$N(2.00, 1.40^2)$	0.488
Normal 5	$N(0.00, 1.00^2)$	$N(0.75, 1.00^2)$	$N(1.50, 1.20^2)$	$N(2.00, 1.40^2)$	0.498
Normal 6	$N(0.00, 1.00^2)$	$N(0.75, 1.00^2)$	$N(1.50, 1.00^2)$	$N(2.00, 1.20^2)$	0.532
Normal 7	$N(0.00, 1.00^2)$	$N(0.00, 1.20^2)$	$N(1.50, 1.20^2)$	$N(1.50, 1.40^2)$	0.548
Normal 8	$N(0.00, 1.00^2)$	$N(0.75, 1.00^2)$	$N(1.50, 1.00^2)$	$N(2.00, 1.00^2)$	0.552
Normal 9	$N(-.50, 1.00^2)$	$N(0.50, 1.00^2)$	$N(1.50, 1.00^2)$	$N(2.00, 1.00^2)$	0.652
Normal 10	$N(0.00, 1.00^2)$	$N(0.00, 1.00^2)$	$N(2.00, 1.00^2)$	$N(2.00, 1.00^2)$	0.775
Gamma 1	$G(1.00, 1.00)$	$G(3.00, 1.732)$	$G(2.00, 1.00)$	$G(4.00, 1.414)$	0.380
Gamma 2	$G(1.00, 0.80)$	$G(1.00, 0.80)$	$G(2.00, 0.50)$	$G(2.00, 0.50)$	0.622
Gamma 3	$G(1.00, 3.00)$	$G(1.00, 2.00)$	$G(2.00, 2.00)$	$G(3.00, 1.50)$	0.636
Gamma 4	$G(1.00, 1.00)$	$G(2.00, 1.414)$	$G(2.00, 0.577)$	$G(3.00, 0.707)$	0.652
Gamma 5	$G(1.00, 2.00)$	$G(2.00, 2.828)$	$G(2.00, 1.00)$	$G(2.00, 0.80)$	0.673
Gamma 6	$G(1.00, 3.00)$	$G(1.50, 2.00)$	$G(2.00, 1.00)$	$G(2.00, 0.80)$	0.697
Gamma 7	$G(1.00, 1.00)$	$G(1.00, 1.00)$	$G(2.00, 0.50)$	$G(2.00, 0.50)$	0.699
Gamma 8	$G(1.00, 2.00)$	$G(1.50, 2.00)$	$G(2.00, 1.00)$	$G(2.00, 0.50)$	0.711
Gamma 9	$G(2.00, 2.00)$	$G(2.00, 1.00)$	$G(4.00, 1.00)$	$G(4.00, 0.80)$	0.722
Gamma 10	$G(1.00, 3.00)$	$G(1.00, 2.00)$	$G(2.00, 1.00)$	$G(2.00, 0.50)$	0.820

Table 4. Relative bias (RB) of the optimal cut-point estimates under normal distribution.

Scenario	Sample size	Relative bias (RB)						
		YI	MaxA	NWC	MMR	MinA	SEC	PYI
Normal 1	(20,20,20,20)	0.135	0.090	-0.395	0.086	0.065	0.120	0.268
	(20,20,40,40)	0.154	0.105	-0.393	0.103	0.084	0.136	0.256
	(30,30,30,30)	0.138	0.085	-0.431	0.077	0.057	0.107	0.231
	(50,50,50,50)	0.138	0.078	-0.471	0.068	0.051	0.098	0.217
	(75,75,50,50)	0.134	0.072	-0.490	0.059	0.043	0.091	0.202
	(75,75,75,75)	0.136	0.079	-0.493	0.068	0.053	0.096	0.206
	(100,100,100,100)	0.131	0.075	-0.506	0.066	0.050	0.092	0.198
Normal 2	(20,20,20,20)	0.001	0.016	0.020	0.018	0.016	0.016	0.043
	(20,20,40,40)	0.010	0.022	0.023	0.028	0.027	0.026	0.039
	(30,30,30,30)	0.002	0.011	0.012	0.011	0.009	0.011	0.030
	(50,50,50,50)	0.000	0.003	0.002	0.003	0.003	0.004	0.016
	(75,75,50,50)	-0.003	-0.004	-0.004	-0.003	-0.003	-0.005	0.006
	(75,75,75,75)	0.001	0.004	0.003	0.004	0.004	0.003	0.016
	(100,100,100,100)	0.001	0.002	0.000	0.003	0.003	0.002	0.009
Normal 3	(20,20,20,20)	0.391	0.024	0.024	0.022	0.018	0.020	0.008
	(20,20,40,40)	0.403	0.031	0.034	0.039	0.039	0.040	0.016
	(30,30,30,30)	0.393	0.015	0.016	0.014	0.011	0.012	0.005
	(50,50,50,50)	0.385	0.009	0.010	0.009	0.007	0.008	0.004
	(75,75,50,50)	0.379	0.000	0.001	-0.002	-0.003	-0.002	-0.008
	(75,75,75,75)	0.385	0.007	0.008	0.009	0.009	0.009	0.004
	(100,100,100,100)	0.378	0.005	0.005	0.004	0.004	0.004	0.003
Normal 4	(20,20,20,20)	0.171	0.026	0.033	0.026	0.021	0.026	0.050
	(20,20,40,40)	0.105	0.029	0.037	0.035	0.034	0.037	0.038
	(30,30,30,30)	0.121	0.014	0.021	0.014	0.012	0.016	0.022
	(50,50,50,50)	0.069	0.008	0.013	0.009	0.007	0.010	0.018
	(75,75,50,50)	0.060	0.000	0.007	-0.001	-0.002	0.000	0.012
	(75,75,75,75)	0.042	0.006	0.013	0.007	0.008	0.008	0.016
	(100,100,100,100)	0.029	0.005	0.009	0.005	0.003	0.006	0.012
Normal 5	(20,20,20,20)	0.190	0.028	0.034	0.025	0.020	0.024	0.045
	(20,20,40,40)	0.129	0.032	0.044	0.044	0.042	0.044	0.041
	(30,30,30,30)	0.147	0.013	0.018	0.003	-0.001	0.004	0.029
	(50,50,50,50)	0.097	0.008	0.014	0.007	0.007	0.009	0.015
	(75,75,50,50)	0.089	-0.003	0.002	-0.004	-0.004	-0.003	0.003
	(75,75,75,75)	0.066	0.006	0.012	0.008	0.007	0.008	0.014
	(100,100,100,100)	0.054	0.005	0.010	0.005	0.005	0.006	0.013
Normal 6	(20,20,20,20)	0.200	0.042	0.042	0.042	0.040	0.042	0.059
	(20,20,40,40)	0.163	0.061	0.057	0.056	0.051	0.056	0.088
	(30,30,30,30)	0.204	0.031	0.033	0.021	0.015	0.021	0.064
	(50,50,50,50)	0.189	0.036	0.033	0.032	0.030	0.036	0.067
	(75,75,50,50)	0.201	0.027	0.026	0.013	0.011	0.016	0.057
	(75,75,75,75)	0.189	0.028	0.030	0.021	0.018	0.025	0.060
	(100,100,100,100)	0.151	0.029	0.027	0.025	0.023	0.028	0.054
Normal 7	(20,20,20,20)	0.341	0.138	0.131	0.108	0.098	0.111	0.202
	(20,20,40,40)	0.282	0.140	0.141	0.128	0.120	0.131	0.190
	(30,30,30,30)	0.306	0.122	0.116	0.095	0.086	0.102	0.168
	(50,50,50,50)	0.277	0.115	0.109	0.083	0.077	0.089	0.169
	(75,75,50,50)	0.279	0.105	0.097	0.067	0.061	0.074	0.156
	(75,75,75,75)	0.265	0.114	0.105	0.078	0.072	0.084	0.165
	(100,100,100,100)	0.260	0.109	0.101	0.074	0.068	0.080	0.161
Normal 8	(20,20,20,20)	0.006	0.024	0.023	0.021	0.018	0.018	0.016
	(20,20,40,40)	-0.036	0.029	0.032	0.038	0.037	0.036	0.019
	(30,30,30,30)	-0.004	0.013	0.013	0.011	0.008	0.009	0.006
	(50,50,50,50)	-0.013	0.008	0.009	0.008	0.008	0.008	0.004
	(75,75,50,50)	0.01	0.001	-0.001	-0.005	-0.005	-0.005	-0.003
	(75,75,75,75)	0.000	0.005	0.006	0.006	0.006	0.006	0.006
	(100,100,100,100)	-0.006	0.004	0.004	0.005	0.004	0.005	0.001
Normal 9	(20,20,20,20)	0.015	0.034	0.035	0.037	0.036	0.037	0.025
	(20,20,40,40)	0.002	0.041	0.047	0.066	0.066	0.066	0.029
	(30,30,30,30)	0.013	0.018	0.018	0.029	0.023	0.024	0.015
	(50,50,50,50)	0.003	0.013	0.012	0.015	0.014	0.014	0.015
	(75,75,50,50)	-0.002	-0.003	-0.004	-0.005	-0.005	-0.005	-0.006
	(75,75,75,75)	0.006	0.007	0.007	0.012	0.011	0.012	0.007
	(100,100,100,100)	0.001	0.008	0.007	0.009	0.008	0.009	0.009
Normal 10	(20,20,20,20)	0.035	0.032	0.031	0.037	0.034	0.036	0.041
	(20,20,40,40)	0.023	0.031	0.034	0.049	0.048	0.047	0.037
	(30,30,30,30)	0.022	0.020	0.019	0.024	0.020	0.020	0.027
	(50,50,50,50)	0.014	0.014	0.015	0.015	0.015	0.015	0.019
	(75,75,50,50)	0.009	0.004	0.002	0.000	0.000	0.000	0.004
	(75,75,75,75)	0.007	0.008	0.008	0.008	0.007	0.007	0.012
	(100,100,100,100)	0.008	0.007	0.006	0.006	0.005	0.005	0.011

Table 5. Root mean squared error (RMSE) of the optimal cut-point estimate under normal distribution.

Scenario	Sample size	Root mean squared error (RMSE)						
		YI	MaxA	NWC	MMR	MinA	SEC	PYI
Normal 1	(20,20,20,20)	0.347	0.380	0.725	0.331	0.295	0.398	0.826
	(20,20,40,40)	0.354	0.360	0.709	0.319	0.281	0.391	0.782
	(30,30,30,30)	0.334	0.336	0.753	0.291	0.253	0.356	0.740
	(50,50,50,50)	0.315	0.289	0.777	0.247	0.212	0.311	0.673
	(75,75,50,50)	0.308	0.269	0.791	0.228	0.196	0.293	0.632
	(75,75,75,75)	0.303	0.260	0.786	0.222	0.188	0.285	0.625
	(100,100,100,100)	0.290	0.240	0.791	0.205	0.171	0.265	0.584
Normal 2	(20,20,20,20)	0.217	0.295	0.456	0.254	0.233	0.294	0.523
	(20,20,40,40)	0.201	0.270	0.417	0.232	0.211	0.267	0.488
	(30,30,30,30)	0.196	0.257	0.405	0.217	0.196	0.254	0.481
	(50,50,50,50)	0.176	0.215	0.340	0.181	0.162	0.213	0.409
	(75,75,50,50)	0.168	0.202	0.323	0.169	0.149	0.201	0.391
	(75,75,75,75)	0.163	0.186	0.303	0.156	0.138	0.186	0.368
	(100,100,100,100)	0.157	0.166	0.277	0.140	0.123	0.168	0.336
Normal 3	(20,20,20,20)	0.895	0.301	0.300	0.252	0.244	0.258	0.398
	(20,20,40,40)	0.910	0.277	0.273	0.232	0.226	0.239	0.370
	(30,30,30,30)	0.890	0.264	0.262	0.211	0.205	0.218	0.360
	(50,50,50,50)	0.874	0.220	0.219	0.174	0.168	0.180	0.309
	(75,75,50,50)	0.863	0.207	0.206	0.161	0.156	0.167	0.291
	(75,75,75,75)	0.870	0.190	0.192	0.150	0.145	0.155	0.273
	(100,100,100,100)	0.859	0.172	0.173	0.134	0.129	0.139	0.250
Normal 4	(20,20,20,20)	0.896	0.314	0.307	0.259	0.250	0.269	0.408
	(20,20,40,40)	0.723	0.284	0.278	0.235	0.226	0.246	0.374
	(30,30,30,30)	0.778	0.269	0.263	0.219	0.211	0.229	0.355
	(50,50,50,50)	0.652	0.226	0.220	0.179	0.172	0.188	0.305
	(75,75,50,50)	0.633	0.214	0.206	0.165	0.158	0.174	0.286
	(75,75,75,75)	0.589	0.194	0.189	0.152	0.147	0.161	0.261
	(100,100,100,100)	0.517	0.176	0.172	0.137	0.131	0.145	0.244
Normal 5	(20,20,20,20)	0.872	0.317	0.307	0.264	0.257	0.273	0.411
	(20,20,40,40)	0.749	0.288	0.280	0.240	0.233	0.248	0.375
	(30,30,30,30)	0.794	0.265	0.257	0.216	0.212	0.223	0.356
	(50,50,50,50)	0.663	0.230	0.221	0.184	0.178	0.191	0.313
	(75,75,50,50)	0.663	0.216	0.207	0.171	0.165	0.177	0.294
	(75,75,75,75)	0.598	0.201	0.193	0.157	0.151	0.163	0.274
	(100,100,100,100)	0.548	0.181	0.174	0.141	0.136	0.147	0.249
Normal 6	(20,20,20,20)	0.800	0.292	0.276	0.241	0.235	0.251	0.368
	(20,20,40,40)	0.742	0.282	0.264	0.226	0.220	0.232	0.370
	(30,30,30,30)	0.763	0.255	0.236	0.204	0.199	0.206	0.341
	(50,50,50,50)	0.680	0.224	0.209	0.181	0.175	0.186	0.306
	(75,75,50,50)	0.689	0.203	0.189	0.154	0.152	0.162	0.271
	(75,75,75,75)	0.645	0.188	0.175	0.150	0.146	0.157	0.271
	(100,100,100,100)	0.601	0.170	0.158	0.133	0.129	0.139	0.244
Normal 7	(20,20,20,20)	0.932	0.371	0.346	0.296	0.286	0.307	0.488
	(20,20,40,40)	0.811	0.344	0.327	0.284	0.273	0.294	0.451
	(30,30,30,30)	0.838	0.324	0.300	0.254	0.244	0.267	0.422
	(50,50,50,50)	0.737	0.278	0.256	0.210	0.202	0.220	0.383
	(75,75,50,50)	0.722	0.259	0.236	0.190	0.182	0.200	0.356
	(75,75,75,75)	0.683	0.252	0.230	0.185	0.176	0.195	0.351
	(100,100,100,100)	0.639	0.231	0.212	0.168	0.159	0.177	0.329
Normal 8	(20,20,20,20)	0.627	0.282	0.263	0.231	0.225	0.237	0.346
	(20,20,40,40)	0.599	0.259	0.239	0.212	0.208	0.216	0.320
	(30,30,30,30)	0.586	0.244	0.224	0.195	0.190	0.199	0.310
	(50,50,50,50)	0.521	0.205	0.188	0.160	0.156	0.164	0.264
	(75,75,50,50)	0.503	0.193	0.176	0.150	0.146	0.154	0.251
	(75,75,75,75)	0.481	0.179	0.164	0.138	0.134	0.142	0.233
	(100,100,100,100)	0.426	0.161	0.147	0.123	0.119	0.127	0.214
Normal 9	(20,20,20,20)	0.447	0.283	0.250	0.242	0.238	0.247	0.332
	(20,20,40,40)	0.409	0.261	0.230	0.233	0.230	0.234	0.304
	(30,30,30,30)	0.397	0.245	0.213	0.206	0.202	0.208	0.295
	(50,50,50,50)	0.344	0.205	0.176	0.167	0.164	0.169	0.248
	(75,75,50,50)	0.324	0.191	0.162	0.150	0.149	0.152	0.234
	(75,75,75,75)	0.306	0.178	0.151	0.141	0.139	0.143	0.220
	(100,100,100,100)	0.281	0.162	0.137	0.126	0.124	0.128	0.200
Normal 10	(20,20,20,20)	0.355	0.268	0.232	0.228	0.224	0.234	0.300
	(20,20,40,40)	0.316	0.246	0.210	0.209	0.208	0.211	0.270
	(30,30,30,30)	0.305	0.227	0.193	0.188	0.185	0.190	0.255
	(50,50,50,50)	0.260	0.192	0.160	0.153	0.151	0.155	0.216
	(75,75,50,50)	0.243	0.180	0.147	0.139	0.138	0.140	0.203
	(75,75,75,75)	0.230	0.165	0.134	0.128	0.126	0.129	0.189
	(100,100,100,100)	0.211	0.150	0.121	0.114	0.112	0.115	0.175

Table 6. Relative bias (RB) of the optimal cut-point estimate under gamma distribution.

Scenario	Sample size	Relative bias (RB)						
		YI	MaxA	NWC	MMR	MinA	SEC	PYI
Gamma 1	(20,20,20,20)	-0.426	0.009	0.011	0.005	0.004	0.001	0.011
	(20,20,40,40)	-0.833	0.019	0.023	0.024	0.025	0.022	0.023
	(30,30,30,30)	-0.604	0.006	0.008	0.004	0.004	0.001	0.000
	(50,50,50,50)	-0.910	0.001	0.002	0.002	0.002	0.000	0.006
	(75,75,50,50)	-0.781	-0.004	-0.001	-0.008	-0.008	-0.009	-0.003
	(75,75,75,75)	-1.229	0.001	0.005	0.000	0.000	-0.001	0.006
	(100,100,100,100)	-1.456	-0.001	0.000	-0.002	-0.002	-0.004	-0.002
Gamma 2	(20,20,20,20)	-0.030	0.001	0.006	0.012	0.010	0.009	-0.004
	(20,20,40,40)	-0.035	0.012	0.020	0.032	0.031	0.030	0.006
	(30,30,30,30)	-0.033	-0.002	0.000	0.005	0.004	0.003	-0.013
	(50,50,50,50)	-0.023	-0.001	0.001	0.003	0.002	0.002	0.000
	(75,75,75,75)	-0.021	-0.002	0.000	0.000	0.001	0.000	-0.005
	(75,75,50,50)	-0.019	-0.011	-0.009	-0.009	-0.009	-0.009	-0.013
	(100,100,100,100)	-0.017	-0.002	-0.001	0.001	0.001	0.000	-0.001
Gamma 3	(20,20,20,20)	-0.038	-0.005	0.000	0.001	0.000	-0.002	0.004
	(20,20,40,40)	-0.043	0.012	0.018	0.032	0.032	0.030	0.012
	(30,30,30,30)	-0.038	-0.005	-0.002	-0.003	-0.004	-0.007	-0.009
	(50,50,50,50)	-0.033	-0.007	-0.004	-0.003	-0.003	-0.004	-0.005
	(75,75,50,50)	-0.027	-0.013	-0.010	-0.016	-0.016	-0.017	-0.012
	(75,75,75,75)	-0.026	-0.006	-0.003	-0.004	-0.004	-0.004	-0.004
	(100,100,100,100)	-0.024	-0.006	-0.004	-0.006	-0.006	-0.006	-0.008
Gamma 4	(20,20,20,20)	0.019	0.008	0.009	0.012	0.011	0.010	0.017
	(20,20,40,40)	0.015	0.017	0.024	0.030	0.030	0.029	0.019
	(30,30,30,30)	0.012	0.005	0.007	0.007	0.005	0.005	0.008
	(50,50,50,50)	0.010	0.002	0.004	0.003	0.003	0.003	0.008
	(75,75,50,50)	0.008	-0.005	-0.002	-0.006	-0.006	-0.006	-0.002
	(75,75,75,75)	0.004	0.000	0.003	0.002	0.001	0.002	0.001
	(100,100,100,100)	0.003	0.000	0.002	0.000	0.000	0.000	0.000
Gamma 5	(20,20,20,20)	0.041	0.014	0.014	0.018	0.017	0.016	0.028
	(20,20,40,40)	0.032	0.021	0.025	0.035	0.034	0.035	0.030
	(30,30,30,30)	0.022	0.009	0.010	0.012	0.009	0.009	0.015
	(50,50,50,50)	0.017	0.004	0.007	0.005	0.004	0.005	0.012
	(75,75,50,50)	0.016	-0.005	-0.002	-0.006	-0.006	-0.006	0.002
	(75,75,75,75)	0.011	0.001	0.003	0.003	0.002	0.002	0.007
	(100,100,100,100)	0.010	0.002	0.004	0.003	0.002	0.003	0.008
Gamma 6	(20,20,20,20)	0.036	0.011	0.014	0.030	0.029	0.029	0.015
	(20,20,40,40)	0.020	0.020	0.026	0.044	0.043	0.045	0.018
	(30,30,30,30)	0.019	0.007	0.011	0.020	0.017	0.018	0.008
	(50,50,50,50)	0.007	0.002	0.006	0.010	0.009	0.010	0.005
	(75,75,50,50)	0.009	-0.007	-0.003	-0.002	-0.003	-0.002	-0.007
	(75,75,75,75)	0.009	0.001	0.005	0.009	0.008	0.009	0.001
	(100,100,100,100)	0.009	0.002	0.004	0.006	0.006	0.006	0.002
Gamma 7	(20,20,20,20)	-0.001	0.007	0.010	0.015	0.014	0.012	0.011
	(20,20,40,40)	0.001	0.017	0.025	0.035	0.034	0.034	0.018
	(30,30,30,30)	-0.005	0.003	0.006	0.009	0.007	0.006	0.004
	(50,50,50,50)	-0.002	0.000	0.002	0.003	0.003	0.002	0.007
	(75,75,50,50)	-0.002	-0.009	-0.006	-0.007	-0.008	-0.007	-0.008
	(75,75,75,75)	-0.005	0.000	0.002	0.003	0.003	0.003	0.000
	(100,100,100,100)	-0.004	-0.001	0.001	0.002	0.001	0.001	0.001
Gamma 8	(20,20,20,20)	0.024	0.011	0.010	0.013	0.014	0.010	0.024
	(20,20,40,40)	0.017	0.015	0.023	0.036	0.036	0.035	0.025
	(30,30,30,30)	0.009	0.003	0.007	0.006	0.003	0.002	0.010
	(50,50,50,50)	0.005	-0.003	0.002	-0.001	-0.001	-0.002	0.007
	(75,75,50,50)	0.003	-0.009	-0.008	-0.012	-0.012	-0.012	-0.004
	(75,75,75,75)	0.003	0.000	0.001	0.000	0.000	-0.001	0.005
	(100,100,100,100)	0.003	-0.001	0.001	0.000	-0.001	0.000	0.003
Gamma 9	(20,20,20,20)	-0.008	0.011	0.014	0.038	0.039	0.040	0.007
	(20,20,40,40)	-0.004	0.019	0.023	0.051	0.051	0.052	0.012
	(30,30,30,30)	-0.008	0.007	0.009	0.024	0.020	0.021	0.004
	(50,50,50,50)	-0.010	0.003	0.005	0.013	0.013	0.014	0.002
	(75,75,50,50)	-0.009	-0.002	-0.002	0.002	0.002	0.002	-0.006
	(75,75,75,75)	-0.006	0.001	0.001	0.007	0.007	0.006	-0.001
	(100,100,100,100)	-0.007	0.001	0.001	0.006	0.006	0.006	0.001
Gamma 10	(20,20,20,20)	0.014	0.020	0.016	0.034	0.040	0.033	0.033
	(20,20,40,40)	0.025	0.030	0.035	0.053	0.055	0.053	0.037
	(30,30,30,30)	0.008	0.008	0.007	0.018	0.015	0.009	0.020
	(50,50,50,50)	0.004	0.004	0.004	0.003	0.004	0.001	0.015
	(75,75,50,50)	0.003	0.004	0.002	0.003	0.002	0.001	0.010
	(75,75,75,75)	0.000	-0.008	-0.005	-0.010	-0.010	-0.010	-0.002
	(100,100,100,100)	0.001	0.001	0.002	0.002	0.002	0.001	0.008

Table 7. Root mean squared error (RMSE) of the optimal cut-point estimate under gamma distribution.

Scenario	Sample size	Root mean squared error (RMSE)						
		YI	MaxA	NWC	MMR	MinA	SEC	PYI
Gamma 1	(20,20,20,20)	1.662	0.328	0.375	0.268	0.259	0.280	0.479
	(20,20,40,40)	2.157	0.301	0.352	0.246	0.236	0.257	0.453
	(30,30,30,30)	1.993	0.289	0.328	0.229	0.220	0.243	0.437
	(50,50,50,50)	2.550	0.239	0.281	0.194	0.183	0.206	0.380
	(75,75,50,50)	2.546	0.229	0.268	0.180	0.171	0.191	0.356
	(75,75,75,75)	3.079	0.209	0.248	0.166	0.156	0.176	0.340
	(100,100,100,100)	3.470	0.188	0.225	0.149	0.141	0.159	0.311
Gamma 2	(20,20,20,20)	0.816	0.443	0.392	0.357	0.351	0.368	0.535
	(20,20,40,40)	0.778	0.406	0.360	0.324	0.318	0.331	0.492
	(30,30,30,30)	0.749	0.389	0.343	0.304	0.298	0.314	0.474
	(50,50,50,50)	0.643	0.321	0.282	0.250	0.244	0.257	0.406
	(75,75,50,50)	0.605	0.310	0.267	0.236	0.230	0.242	0.384
	(75,75,75,75)	0.577	0.283	0.246	0.215	0.210	0.222	0.356
	(100,100,100,100)	0.532	0.258	0.221	0.194	0.188	0.200	0.326
Gamma 3	(20,20,20,20)	0.259	0.154	0.134	0.138	0.135	0.141	0.186
	(20,20,40,40)	0.245	0.138	0.121	0.123	0.122	0.125	0.170
	(30,30,30,30)	0.232	0.133	0.117	0.116	0.115	0.119	0.166
	(50,50,50,50)	0.202	0.112	0.096	0.095	0.094	0.096	0.142
	(75,75,50,50)	0.187	0.108	0.093	0.092	0.091	0.093	0.136
	(75,75,75,75)	0.178	0.098	0.084	0.080	0.079	0.082	0.124
	(100,100,100,100)	0.164	0.088	0.076	0.073	0.072	0.074	0.115
Gamma 4	(20,20,20,20)	0.642	0.391	0.341	0.321	0.313	0.330	0.458
	(20,20,40,40)	0.602	0.353	0.305	0.286	0.282	0.290	0.422
	(30,30,30,30)	0.586	0.341	0.293	0.274	0.271	0.280	0.408
	(50,50,50,50)	0.499	0.286	0.241	0.222	0.218	0.227	0.347
	(75,75,50,50)	0.474	0.275	0.228	0.212	0.208	0.215	0.333
	(75,75,75,75)	0.449	0.253	0.211	0.190	0.187	0.194	0.313
	(100,100,100,100)	0.406	0.230	0.191	0.172	0.168	0.175	0.286
Gamma 5	(20,20,20,20)	0.337	0.214	0.184	0.175	0.172	0.179	0.254
	(20,20,40,40)	0.313	0.195	0.166	0.160	0.157	0.162	0.231
	(30,30,30,30)	0.298	0.183	0.154	0.145	0.143	0.149	0.218
	(50,50,50,50)	0.259	0.157	0.129	0.121	0.119	0.123	0.192
	(75,75,50,50)	0.246	0.150	0.122	0.114	0.112	0.116	0.181
	(75,75,75,75)	0.230	0.137	0.112	0.102	0.100	0.104	0.169
	(100,100,100,100)	0.213	0.124	0.101	0.092	0.090	0.094	0.155
Gamma 6	(20,20,20,20)	0.325	0.217	0.185	0.173	0.170	0.177	0.250
	(20,20,40,40)	0.303	0.200	0.169	0.167	0.166	0.169	0.231
	(30,30,30,30)	0.292	0.188	0.158	0.147	0.144	0.148	0.219
	(50,50,50,50)	0.250	0.158	0.131	0.118	0.117	0.120	0.185
	(75,75,50,50)	0.234	0.153	0.125	0.111	0.109	0.112	0.180
	(75,75,75,75)	0.224	0.139	0.114	0.102	0.100	0.103	0.167
	(100,100,100,100)	0.206	0.126	0.101	0.090	0.089	0.091	0.150
Gamma 7	(20,20,20,20)	0.600	0.390	0.336	0.317	0.312	0.325	0.449
	(20,20,40,40)	0.559	0.362	0.306	0.289	0.286	0.294	0.417
	(30,30,30,30)	0.543	0.347	0.292	0.270	0.266	0.277	0.400
	(50,50,50,50)	0.463	0.290	0.241	0.223	0.220	0.228	0.340
	(75,75,50,50)	0.432	0.279	0.228	0.209	0.206	0.213	0.328
	(75,75,75,75)	0.415	0.254	0.209	0.191	0.187	0.194	0.303
	(100,100,100,100)	0.380	0.231	0.188	0.169	0.166	0.173	0.277
Gamma 8	(20,20,20,20)	0.367	0.249	0.210	0.218	0.214	0.223	0.296
	(20,20,40,40)	0.339	0.229	0.191	0.198	0.196	0.200	0.273
	(30,30,30,30)	0.330	0.220	0.182	0.185	0.183	0.190	0.260
	(50,50,50,50)	0.283	0.186	0.149	0.150	0.149	0.153	0.223
	(75,75,50,50)	0.268	0.175	0.142	0.144	0.143	0.146	0.213
	(75,75,75,75)	0.254	0.161	0.131	0.127	0.125	0.129	0.197
	(100,100,100,100)	0.230	0.147	0.117	0.115	0.114	0.117	0.181
Gamma 9	(20,20,20,20)	0.542	0.404	0.350	0.352	0.348	0.357	0.445
	(20,20,40,40)	0.492	0.368	0.319	0.337	0.334	0.340	0.400
	(30,30,30,30)	0.483	0.350	0.300	0.290	0.285	0.294	0.390
	(50,50,50,50)	0.411	0.297	0.248	0.230	0.227	0.232	0.325
	(75,75,50,50)	0.389	0.283	0.235	0.211	0.209	0.213	0.315
	(75,75,75,75)	0.365	0.257	0.212	0.197	0.195	0.198	0.294
	(100,100,100,100)	0.333	0.234	0.191	0.173	0.171	0.174	0.263
Gamma 10	(20,20,20,20)	0.268	0.209	0.177	0.195	0.193	0.199	0.236
	(20,20,40,40)	0.241	0.194	0.161	0.180	0.179	0.182	0.218
	(30,30,30,30)	0.233	0.183	0.151	0.158	0.156	0.162	0.205
	(50,50,50,50)	0.202	0.157	0.123	0.130	0.128	0.132	0.176
	(75,75,50,50)	0.179	0.136	0.105	0.109	0.108	0.110	0.156
	(75,75,75,75)	0.186	0.151	0.117	0.123	0.123	0.124	0.167
	(100,100,100,100)	0.162	0.124	0.094	0.097	0.097	0.098	0.140

Table 8. Total correct classification rate (TCCR) estimates under normal distribution.

Scenario	Sample size	Total CCR						
		YI	MaxA	NWC	MMR	MinA	SEC	PYI
Normal 1	(20,20,20,20)	0.995	0.787	0.842	0.766	0.757	0.783	0.858
	(20,20,40,40)	1.003	0.774	0.824	0.756	0.747	0.771	0.846
	(30,30,30,30)	0.995	0.767	0.818	0.752	0.744	0.767	0.838
	(50,50,50,50)	0.997	0.749	0.792	0.737	0.730	0.750	0.809
	(75,75,50,50)	0.996	0.743	0.784	0.733	0.726	0.744	0.801
	(75,75,75,75)	0.998	0.736	0.775	0.727	0.720	0.737	0.792
	(100,100,100,100)	0.998	0.730	0.767	0.721	0.716	0.731	0.782
Normal 2	(20,20,20,20)	1.002	0.817	0.855	0.799	0.791	0.812	0.873
	(20,20,40,40)	1.003	0.805	0.839	0.789	0.782	0.800	0.857
	(30,30,30,30)	0.999	0.798	0.831	0.784	0.777	0.794	0.850
	(50,50,50,50)	0.998	0.778	0.804	0.768	0.763	0.776	0.818
	(75,75,50,50)	0.998	0.771	0.796	0.763	0.758	0.771	0.810
	(75,75,75,75)	0.998	0.765	0.787	0.758	0.753	0.764	0.799
	(100,100,100,100)	0.999	0.757	0.777	0.752	0.748	0.757	0.787
Normal 3	(20,20,20,20)	1.074	1.079	1.078	1.044	1.042	1.046	1.095
	(20,20,40,40)	1.064	1.066	1.065	1.032	1.030	1.034	1.081
	(30,30,30,30)	1.059	1.059	1.059	1.032	1.030	1.034	1.073
	(50,50,50,50)	1.042	1.037	1.036	1.017	1.015	1.018	1.048
	(75,75,50,50)	1.036	1.030	1.030	1.013	1.012	1.014	1.041
	(75,75,75,75)	1.030	1.023	1.023	1.007	1.006	1.008	1.032
	(100,100,100,100)	1.023	1.016	1.016	1.002	1.001	1.003	1.023
Normal 4	(20,20,20,20)	1.148	1.098	1.095	1.072	1.070	1.077	1.118
	(20,20,40,40)	1.138	1.088	1.084	1.063	1.060	1.067	1.109
	(30,30,30,30)	1.128	1.081	1.079	1.061	1.058	1.065	1.101
	(50,50,50,50)	1.107	1.062	1.060	1.046	1.043	1.049	1.080
	(75,75,50,50)	1.099	1.057	1.055	1.042	1.040	1.045	1.073
	(75,75,75,75)	1.093	1.051	1.049	1.037	1.034	1.039	1.065
	(100,100,100,100)	1.085	1.044	1.043	1.032	1.030	1.034	1.059
Normal 5	(20,20,20,20)	1.161	1.117	1.113	1.081	1.079	1.084	1.133
	(20,20,40,40)	1.149	1.106	1.101	1.069	1.067	1.072	1.121
	(30,30,30,30)	1.137	1.097	1.093	1.068	1.067	1.071	1.111
	(50,50,50,50)	1.116	1.078	1.076	1.055	1.053	1.057	1.091
	(75,75,50,50)	1.108	1.072	1.070	1.051	1.049	1.053	1.084
	(75,75,75,75)	1.100	1.065	1.063	1.045	1.043	1.046	1.076
	(100,100,100,100)	1.092	1.058	1.056	1.040	1.039	1.042	1.068
Normal 6	(20,20,20,20)	1.182	1.151	1.146	1.119	1.117	1.122	1.161
	(20,20,40,40)	1.171	1.143	1.139	1.114	1.112	1.115	1.155
	(30,30,30,30)	1.163	1.137	1.132	1.112	1.111	1.114	1.146
	(50,50,50,50)	1.137	1.116	1.113	1.099	1.098	1.100	1.123
	(75,75,50,50)	1.131	1.112	1.110	1.097	1.097	1.098	1.118
	(75,75,75,75)	1.124	1.104	1.102	1.090	1.090	1.092	1.111
	(100,100,100,100)	1.111	1.094	1.093	1.083	1.082	1.084	1.100
Normal 7	(20,20,20,20)	1.204	1.174	1.168	1.151	1.149	1.154	1.185
	(20,20,40,40)	1.192	1.164	1.158	1.142	1.140	1.145	1.176
	(30,30,30,30)	1.184	1.157	1.152	1.140	1.138	1.142	1.168
	(50,50,50,50)	1.162	1.139	1.135	1.127	1.125	1.128	1.149
	(75,75,50,50)	1.155	1.135	1.131	1.124	1.122	1.125	1.144
	(75,75,75,75)	1.148	1.128	1.125	1.118	1.117	1.120	1.136
	(100,100,100,100)	1.140	1.122	1.120	1.114	1.113	1.115	1.130
Normal 8	(20,20,20,20)	1.210	1.184	1.179	1.153	1.151	1.154	1.193
	(20,20,40,40)	1.194	1.171	1.166	1.143	1.142	1.144	1.180
	(30,30,30,30)	1.188	1.165	1.160	1.142	1.140	1.143	1.173
	(50,50,50,50)	1.162	1.145	1.141	1.128	1.127	1.129	1.151
	(75,75,50,50)	1.155	1.138	1.135	1.124	1.123	1.124	1.145
	(75,75,75,75)	1.147	1.132	1.129	1.119	1.118	1.120	1.138
	(100,100,100,100)	1.138	1.125	1.123	1.114	1.114	1.115	1.130
Normal 9	(20,20,20,20)	1.337	1.325	1.317	1.286	1.285	1.287	1.330
	(20,20,40,40)	1.324	1.313	1.306	1.277	1.277	1.278	1.318
	(30,30,30,30)	1.317	1.307	1.300	1.279	1.278	1.280	1.311
	(50,50,50,50)	1.295	1.287	1.282	1.268	1.267	1.268	1.290
	(75,75,50,50)	1.288	1.281	1.276	1.265	1.264	1.265	1.284
	(75,75,75,75)	1.280	1.274	1.270	1.259	1.259	1.259	1.277
	(100,100,100,100)	1.273	1.268	1.264	1.255	1.255	1.256	1.270
Normal 10	(20,20,20,20)	1.493	1.487	1.479	1.470	1.469	1.471	1.489
	(20,20,40,40)	1.485	1.481	1.473	1.465	1.464	1.466	1.483
	(30,30,30,30)	1.479	1.475	1.468	1.462	1.461	1.463	1.476
	(50,50,50,50)	1.464	1.461	1.456	1.452	1.452	1.453	1.462
	(75,75,50,50)	1.460	1.458	1.453	1.450	1.450	1.450	1.459
	(75,75,75,75)	1.455	1.452	1.448	1.446	1.445	1.446	1.453
	(100,100,100,100)	1.449	1.446	1.443	1.441	1.441	1.442	1.448

Table 9. Maximum minimum difference (MMDIF) estimates under normal distribution.

		MMDIF						
Scenario	Sample size	YI	MaxA	NWC	MMR	MinA	SEC	PYI
Normal 1	(20,20,20,20)	0.160	0.214	0.301	0.170	0.151	0.204	0.381
	(20,20,40,40)	0.143	0.196	0.297	0.156	0.135	0.189	0.377
	(30,30,30,30)	0.142	0.188	0.294	0.152	0.131	0.186	0.379
	(50,50,50,50)	0.125	0.166	0.285	0.134	0.113	0.169	0.348
	(75,75,50,50)	0.120	0.157	0.279	0.129	0.108	0.162	0.339
	(75,75,75,75)	0.119	0.148	0.274	0.120	0.099	0.153	0.332
	(100,100,100,100)	0.115	0.140	0.270	0.112	0.093	0.147	0.323
Normal 2	(20,20,20,20)	0.161	0.202	0.300	0.164	0.144	0.197	0.367
	(20,20,40,40)	0.145	0.185	0.280	0.149	0.130	0.179	0.343
	(30,30,30,30)	0.145	0.175	0.272	0.142	0.123	0.171	0.345
	(50,50,50,50)	0.131	0.151	0.241	0.122	0.105	0.149	0.295
	(75,75,50,50)	0.126	0.142	0.230	0.115	0.098	0.141	0.282
	(75,75,75,75)	0.125	0.131	0.215	0.106	0.090	0.130	0.265
	(100,100,100,100)	0.121	0.118	0.200	0.096	0.083	0.119	0.245
Normal 3	(20,20,20,20)	0.359	0.196	0.192	0.168	0.160	0.174	0.282
	(20,20,40,40)	0.343	0.183	0.177	0.156	0.151	0.161	0.264
	(30,30,30,30)	0.346	0.176	0.173	0.146	0.140	0.152	0.256
	(50,50,50,50)	0.339	0.149	0.147	0.126	0.121	0.130	0.224
	(75,75,50,50)	0.336	0.142	0.141	0.119	0.115	0.123	0.213
	(75,75,75,75)	0.334	0.130	0.131	0.114	0.111	0.118	0.199
	(100,100,100,100)	0.329	0.119	0.120	0.104	0.101	0.108	0.184
Normal 4	(20,20,20,20)	0.549	0.212	0.201	0.154	0.146	0.166	0.297
	(20,20,40,40)	0.564	0.205	0.193	0.143	0.134	0.154	0.297
	(30,30,30,30)	0.548	0.192	0.182	0.138	0.130	0.149	0.285
	(50,50,50,50)	0.555	0.172	0.164	0.119	0.111	0.130	0.261
	(75,75,50,50)	0.548	0.166	0.158	0.114	0.106	0.124	0.248
	(75,75,75,75)	0.566	0.157	0.151	0.106	0.097	0.116	0.240
	(100,100,100,100)	0.568	0.148	0.143	0.100	0.092	0.109	0.234
Normal 5	(20,20,20,20)	0.522	0.209	0.196	0.154	0.147	0.163	0.286
	(20,20,40,40)	0.529	0.199	0.183	0.136	0.131	0.144	0.275
	(30,30,30,30)	0.511	0.185	0.172	0.128	0.123	0.135	0.257
	(50,50,50,50)	0.510	0.167	0.157	0.113	0.107	0.120	0.237
	(75,75,50,50)	0.505	0.160	0.152	0.106	0.101	0.112	0.228
	(75,75,75,75)	0.512	0.152	0.143	0.096	0.091	0.102	0.215
	(100,100,100,100)	0.514	0.142	0.134	0.088	0.083	0.094	0.203
Normal 6	(20,20,20,20)	0.444	0.191	0.172	0.154	0.146	0.163	0.257
	(20,20,40,40)	0.429	0.180	0.164	0.135	0.131	0.140	0.241
	(30,30,30,30)	0.414	0.167	0.148	0.124	0.119	0.127	0.231
	(50,50,50,50)	0.379	0.145	0.133	0.112	0.108	0.116	0.203
	(75,75,50,50)	0.365	0.135	0.125	0.100	0.098	0.105	0.181
	(75,75,75,75)	0.362	0.128	0.118	0.097	0.093	0.102	0.185
	(100,100,100,100)	0.355	0.115	0.104	0.083	0.080	0.088	0.164
Normal 7	(20,20,20,20)	0.430	0.193	0.173	0.140	0.134	0.150	0.256
	(20,20,40,40)	0.431	0.186	0.164	0.128	0.122	0.136	0.250
	(30,30,30,30)	0.414	0.177	0.157	0.125	0.119	0.133	0.240
	(50,50,50,50)	0.389	0.150	0.133	0.105	0.100	0.112	0.211
	(75,75,50,50)	0.371	0.144	0.128	0.100	0.094	0.106	0.199
	(75,75,75,75)	0.370	0.135	0.120	0.093	0.088	0.099	0.190
	(100,100,100,100)	0.355	0.125	0.111	0.086	0.081	0.092	0.178
Normal 8	(20,20,20,20)	0.409	0.190	0.170	0.150	0.143	0.156	0.252
	(20,20,40,40)	0.388	0.175	0.155	0.136	0.132	0.140	0.232
	(30,30,30,30)	0.382	0.166	0.147	0.129	0.124	0.133	0.227
	(50,50,50,50)	0.345	0.143	0.127	0.109	0.105	0.113	0.197
	(75,75,50,50)	0.337	0.137	0.122	0.103	0.100	0.107	0.189
	(75,75,75,75)	0.321	0.127	0.113	0.097	0.093	0.100	0.175
	(100,100,100,100)	0.292	0.114	0.102	0.087	0.084	0.091	0.161
Normal 9	(20,20,20,20)	0.299	0.176	0.144	0.157	0.152	0.162	0.227
	(20,20,40,40)	0.275	0.165	0.133	0.151	0.149	0.153	0.208
	(30,30,30,30)	0.269	0.156	0.126	0.140	0.135	0.141	0.205
	(50,50,50,50)	0.238	0.133	0.107	0.118	0.116	0.120	0.176
	(75,75,50,50)	0.226	0.127	0.102	0.111	0.110	0.113	0.169
	(75,75,75,75)	0.215	0.118	0.095	0.109	0.107	0.110	0.160
	(100,100,100,100)	0.197	0.108	0.087	0.102	0.100	0.103	0.147
Normal 10	(20,20,20,20)	0.222	0.148	0.115	0.111	0.106	0.118	0.175
	(20,20,40,40)	0.198	0.141	0.105	0.098	0.096	0.101	0.161
	(30,30,30,30)	0.195	0.130	0.098	0.092	0.089	0.097	0.154
	(50,50,50,50)	0.169	0.112	0.084	0.079	0.077	0.081	0.133
	(75,75,50,50)	0.156	0.109	0.081	0.074	0.072	0.075	0.127
	(75,75,75,75)	0.150	0.098	0.073	0.067	0.065	0.069	0.118
	(100,100,100,100)	0.138	0.090	0.067	0.061	0.060	0.063	0.111

Table 10. Total correct classification rate (TCCR) estimates under gamma distribution.

Scenario	Sample size	Total CCR						
		YI	MaxA	NWC	MMR	MinA	SEC	PYI
Gamma 1	(20,20,20,20)	1.060	0.961	0.971	0.925	0.922	0.928	0.989
	(20,20,40,40)	1.045	0.942	0.951	0.908	0.906	0.911	0.969
	(30,30,30,30)	1.037	0.936	0.943	0.906	0.904	0.909	0.960
	(50,50,50,50)	1.018	0.910	0.917	0.888	0.886	0.890	0.929
	(75,75,50,50)	1.013	0.903	0.910	0.883	0.881	0.885	0.921
	(75,75,75,75)	1.009	0.896	0.902	0.879	0.877	0.881	0.913
	(100,100,100,100)	1.005	0.886	0.891	0.871	0.870	0.872	0.900
Gamma 2	(20,20,20,20)	1.298	1.280	1.272	1.259	1.258	1.261	1.287
	(20,20,40,40)	1.279	1.264	1.257	1.246	1.245	1.247	1.271
	(30,30,30,30)	1.274	1.259	1.253	1.244	1.243	1.245	1.265
	(50,50,50,50)	1.250	1.238	1.234	1.228	1.227	1.229	1.243
	(75,75,50,50)	1.244	1.234	1.230	1.224	1.224	1.225	1.239
	(75,75,75,75)	1.235	1.225	1.222	1.218	1.217	1.219	1.230
	(100,100,100,100)	1.226	1.218	1.215	1.212	1.211	1.212	1.222
Gamma 3	(20,20,20,20)	1.322	1.308	1.300	1.268	1.267	1.268	1.313
	(20,20,40,40)	1.304	1.292	1.285	1.262	1.262	1.263	1.298
	(30,30,30,30)	1.297	1.285	1.278	1.254	1.253	1.254	1.289
	(50,50,50,50)	1.273	1.264	1.259	1.240	1.240	1.241	1.267
	(75,75,50,50)	1.268	1.260	1.255	1.236	1.235	1.236	1.263
	(75,75,75,75)	1.260	1.252	1.248	1.233	1.233	1.233	1.255
	(100,100,100,100)	1.251	1.244	1.241	1.228	1.228	1.228	1.246
Gamma 4	(20,20,20,20)	1.344	1.331	1.321	1.303	1.302	1.304	1.336
	(20,20,40,40)	1.330	1.318	1.308	1.292	1.291	1.293	1.323
	(30,30,30,30)	1.321	1.309	1.301	1.288	1.287	1.290	1.314
	(50,50,50,50)	1.299	1.289	1.283	1.274	1.274	1.275	1.293
	(75,75,50,50)	1.293	1.284	1.279	1.271	1.270	1.272	1.288
	(75,75,75,75)	1.285	1.276	1.271	1.264	1.264	1.265	1.280
	(100,100,100,100)	1.276	1.269	1.265	1.259	1.258	1.259	1.272
Gamma 5	(20,20,20,20)	1.377	1.363	1.353	1.334	1.333	1.336	1.369
	(20,20,40,40)	1.362	1.349	1.338	1.320	1.319	1.321	1.355
	(30,30,30,30)	1.354	1.342	1.334	1.321	1.320	1.322	1.347
	(50,50,50,50)	1.334	1.324	1.316	1.307	1.307	1.308	1.328
	(75,75,50,50)	1.326	1.317	1.310	1.302	1.302	1.303	1.322
	(75,75,75,75)	1.320	1.311	1.305	1.297	1.297	1.298	1.316
	(100,100,100,100)	1.313	1.304	1.298	1.291	1.291	1.292	1.308
Gamma 6	(20,20,20,20)	1.404	1.394	1.384	1.344	1.343	1.344	1.397
	(20,20,40,40)	1.388	1.379	1.369	1.330	1.330	1.330	1.382
	(30,30,30,30)	1.381	1.373	1.365	1.338	1.337	1.339	1.375
	(50,50,50,50)	1.360	1.353	1.347	1.328	1.327	1.328	1.355
	(75,75,50,50)	1.353	1.347	1.342	1.326	1.325	1.326	1.350
	(75,75,75,75)	1.347	1.341	1.336	1.320	1.319	1.320	1.342
	(100,100,100,100)	1.338	1.333	1.329	1.315	1.315	1.315	1.334
Gamma 7	(20,20,20,20)	1.399	1.389	1.380	1.368	1.367	1.370	1.393
	(20,20,40,40)	1.383	1.375	1.366	1.356	1.356	1.358	1.379
	(30,30,30,30)	1.378	1.369	1.361	1.354	1.353	1.355	1.372
	(50,50,50,50)	1.356	1.350	1.344	1.340	1.339	1.340	1.352
	(75,75,50,50)	1.351	1.346	1.340	1.336	1.336	1.337	1.348
	(75,75,75,75)	1.343	1.338	1.333	1.330	1.329	1.330	1.340
	(100,100,100,100)	1.335	1.330	1.326	1.324	1.323	1.324	1.332
Gamma 8	(20,20,20,20)	1.426	1.417	1.406	1.383	1.382	1.384	1.421
	(20,20,40,40)	1.410	1.401	1.390	1.373	1.372	1.373	1.406
	(30,30,30,30)	1.402	1.394	1.385	1.372	1.371	1.372	1.398
	(50,50,50,50)	1.383	1.376	1.369	1.361	1.360	1.361	1.380
	(75,75,50,50)	1.376	1.369	1.363	1.356	1.355	1.356	1.374
	(75,75,75,75)	1.369	1.363	1.358	1.351	1.351	1.352	1.367
	(100,100,100,100)	1.361	1.356	1.350	1.346	1.346	1.346	1.359
Gamma 9	(20,20,20,20)	1.442	1.435	1.426	1.386	1.386	1.386	1.437
	(20,20,40,40)	1.428	1.422	1.415	1.379	1.379	1.379	1.425
	(30,30,30,30)	1.420	1.414	1.407	1.383	1.383	1.383	1.416
	(50,50,50,50)	1.398	1.393	1.387	1.373	1.373	1.373	1.395
	(75,75,50,50)	1.393	1.388	1.383	1.373	1.372	1.373	1.391
	(75,75,75,75)	1.386	1.382	1.378	1.368	1.368	1.368	1.384
	(100,100,100,100)	1.379	1.376	1.372	1.364	1.364	1.364	1.377
Gamma 10	(20,20,20,20)	1.582	1.578	1.569	1.541	1.541	1.541	1.579
	(20,20,40,40)	1.569	1.566	1.556	1.536	1.536	1.537	1.567
	(30,30,30,30)	1.565	1.562	1.554	1.539	1.539	1.539	1.563
	(50,50,50,50)	1.546	1.544	1.538	1.529	1.529	1.530	1.545
	(75,75,50,50)	1.535	1.533	1.528	1.522	1.522	1.522	1.534
	(75,75,75,75)	1.541	1.540	1.534	1.527	1.527	1.527	1.541
	(100,100,100,100)	1.527	1.526	1.521	1.517	1.517	1.517	1.526

Table 11. Maximum minimum difference (MMDIF) estimates under gamma distribution.

		MMDIF						
Scenario	Sample size	YI	MaxA	NWC	MMR	MinA	SEC	PYI
	(20,20,20,20)	0.745	0.202	0.230	0.170	0.161	0.181	0.319
	(20,20,40,40)	0.784	0.190	0.219	0.157	0.148	0.168	0.304
	(30,30,30,30)	0.793	0.181	0.206	0.147	0.139	0.158	0.292
Gamma 1	(50,50,50,50)	0.865	0.154	0.181	0.125	0.117	0.136	0.253
	(75,75,50,50)	0.891	0.145	0.172	0.116	0.108	0.125	0.237
	(75,75,75,75)	0.919	0.135	0.161	0.110	0.102	0.120	0.229
	(100,100,100,100)	0.955	0.123	0.149	0.099	0.092	0.107	0.207
	(20,20,20,20)	0.345	0.180	0.149	0.131	0.125	0.138	0.229
	(20,20,40,40)	0.318	0.168	0.140	0.118	0.113	0.122	0.212
	(30,30,30,30)	0.316	0.157	0.130	0.111	0.107	0.117	0.202
Gamma 2	(50,50,50,50)	0.277	0.133	0.111	0.094	0.090	0.099	0.176
	(75,75,50,50)	0.258	0.130	0.107	0.089	0.085	0.092	0.167
	(75,75,75,75)	0.250	0.118	0.098	0.082	0.079	0.086	0.158
	(100,100,100,100)	0.231	0.110	0.091	0.076	0.073	0.079	0.143
	(20,20,20,20)	0.316	0.182	0.149	0.169	0.165	0.174	0.232
	(20,20,40,40)	0.292	0.167	0.136	0.146	0.144	0.149	0.214
	(30,30,30,30)	0.287	0.160	0.130	0.150	0.149	0.155	0.209
Gamma 3	(50,50,50,50)	0.251	0.136	0.110	0.130	0.129	0.132	0.179
	(75,75,50,50)	0.239	0.130	0.106	0.129	0.128	0.131	0.171
	(75,75,75,75)	0.228	0.120	0.098	0.120	0.119	0.121	0.160
	(100,100,100,100)	0.210	0.110	0.090	0.115	0.114	0.116	0.149
	(20,20,20,20)	0.313	0.180	0.145	0.135	0.129	0.143	0.224
	(20,20,40,40)	0.301	0.172	0.134	0.121	0.118	0.124	0.213
	(30,30,30,30)	0.289	0.160	0.129	0.117	0.114	0.122	0.203
Gamma 4	(50,50,50,50)	0.256	0.140	0.111	0.099	0.096	0.102	0.177
	(75,75,50,50)	0.242	0.135	0.106	0.094	0.091	0.096	0.171
	(75,75,75,75)	0.243	0.128	0.100	0.087	0.085	0.090	0.166
	(100,100,100,100)	0.227	0.118	0.092	0.079	0.077	0.082	0.154
	(20,20,20,20)	0.311	0.183	0.144	0.130	0.124	0.139	0.226
	(20,20,40,40)	0.307	0.183	0.140	0.122	0.119	0.126	0.222
	(30,30,30,30)	0.294	0.167	0.130	0.115	0.111	0.121	0.209
Gamma 5	(50,50,50,50)	0.274	0.151	0.114	0.098	0.096	0.102	0.189
	(75,75,50,50)	0.259	0.146	0.111	0.094	0.091	0.096	0.183
	(75,75,75,75)	0.263	0.140	0.106	0.088	0.085	0.092	0.180
	(100,100,100,100)	0.257	0.132	0.099	0.080	0.078	0.083	0.170
	(20,20,20,20)	0.274	0.172	0.134	0.151	0.147	0.157	0.210
	(20,20,40,40)	0.263	0.166	0.126	0.145	0.144	0.147	0.200
	(30,30,30,30)	0.250	0.153	0.120	0.130	0.126	0.131	0.188
Gamma 6	(50,50,50,50)	0.224	0.135	0.103	0.108	0.107	0.109	0.160
	(75,75,50,50)	0.207	0.130	0.100	0.101	0.099	0.101	0.155
	(75,75,75,75)	0.205	0.121	0.092	0.095	0.094	0.096	0.147
	(100,100,100,100)	0.190	0.110	0.083	0.086	0.085	0.086	0.131
	(20,20,20,20)	0.276	0.165	0.132	0.119	0.113	0.126	0.202
	(20,20,40,40)	0.256	0.159	0.122	0.109	0.106	0.113	0.190
	(30,30,30,30)	0.252	0.147	0.113	0.101	0.098	0.106	0.179
Gamma 7	(50,50,50,50)	0.217	0.127	0.099	0.088	0.085	0.091	0.154
	(75,75,50,50)	0.201	0.124	0.095	0.083	0.081	0.085	0.150
	(75,75,75,75)	0.197	0.113	0.087	0.076	0.074	0.079	0.141
	(100,100,100,100)	0.183	0.103	0.080	0.069	0.067	0.072	0.129
	(20,20,20,20)	0.276	0.176	0.136	0.147	0.141	0.155	0.220
	(20,20,40,40)	0.268	0.173	0.129	0.133	0.131	0.137	0.214
	(30,30,30,30)	0.257	0.160	0.121	0.130	0.128	0.137	0.202
Gamma 8	(50,50,50,50)	0.233	0.142	0.105	0.116	0.114	0.119	0.182
	(75,75,50,50)	0.219	0.134	0.101	0.113	0.111	0.114	0.176
	(75,75,75,75)	0.218	0.127	0.095	0.104	0.102	0.106	0.169
	(100,100,100,100)	0.207	0.120	0.088	0.098	0.096	0.100	0.162
	(20,20,20,20)	0.246	0.163	0.126	0.177	0.175	0.182	0.209
	(20,20,40,40)	0.219	0.152	0.117	0.172	0.171	0.174	0.187
	(30,30,30,30)	0.222	0.146	0.112	0.152	0.147	0.152	0.188
Gamma 9	(50,50,50,50)	0.195	0.128	0.096	0.132	0.131	0.133	0.164
	(75,75,50,50)	0.188	0.124	0.093	0.123	0.122	0.123	0.159
	(75,75,75,75)	0.178	0.114	0.085	0.119	0.118	0.119	0.151
	(100,100,100,100)	0.165	0.105	0.078	0.115	0.115	0.115	0.141
	(20,20,20,20)	0.198	0.141	0.104	0.123	0.118	0.130	0.166
	(20,20,40,40)	0.183	0.137	0.096	0.108	0.106	0.111	0.158
	(30,30,30,30)	0.177	0.127	0.091	0.105	0.103	0.112	0.148
Gamma 10	(50,50,50,50)	0.159	0.113	0.079	0.092	0.090	0.095	0.133
	(75,75,50,50)	0.144	0.102	0.071	0.083	0.081	0.084	0.122
	(75,75,75,75)	0.144	0.110	0.077	0.090	0.089	0.090	0.128
	(100,100,100,100)	0.135	0.096	0.066	0.077	0.076	0.078	0.114

Table 12. Coverage probabilities (and mean length) of proposed 95% confidence intervals for optimal cut-points under normality (2000 simulations).

		GI							NPB						
		YI	MaxA	NWC	MMR	MinA	SEC	PYI	YI	MA	NWC	MMR	MinA	SEC	PYI
Scenario	Sample size	Coverage probability													
Normal 1	(20,20,20,20)	0.965	0.955	0.972	0.932	0.930	0.950	0.960	0.853	0.960	0.934	0.949	0.962	0.951	0.952
	(20,20,40,40)	0.965	0.954	0.988	0.955	0.950	0.965	0.955	0.943	0.948	0.939	0.933	0.949	0.928	0.959
	(30,30,30,30)	0.968	0.945	0.981	0.942	0.947	0.950	0.947	0.952	0.964	0.964	0.960	0.963	0.960	0.973
	(50,50,50,50)	0.957	0.960	0.971	0.953	0.953	0.954	0.946	0.952	0.964	0.964	0.957	0.958	0.966	0.970
	(75,75,50,50)	0.958	0.972	0.973	0.959	0.962	0.956	0.948	0.930	0.967	0.973	0.963	0.961	0.969	0.971
	(75,75,75,75)	0.952	0.950	0.974	0.944	0.946	0.947	0.947	0.955	0.968	0.974	0.960	0.961	0.961	0.973
	(100,100,100,100)	0.948	0.954	0.981	0.959	0.962	0.956	0.948	0.926	0.968	0.974	0.969	0.959	0.965	0.969
Normal 2	(20,20,20,20)	0.960	0.950	0.950	0.949	0.950	0.949	0.957	0.950	0.942	0.953	0.940	0.945	0.949	0.956
	(20,20,40,40)	0.949	0.955	0.957	0.952	0.954	0.954	0.947	0.946	0.948	0.955	0.941	0.952	0.948	0.962
	(30,30,30,30)	0.953	0.950	0.953	0.950	0.950	0.951	0.949	0.945	0.960	0.967	0.959	0.961	0.961	0.962
	(50,50,50,50)	0.953	0.956	0.955	0.956	0.957	0.957	0.955	0.931	0.963	0.966	0.956	0.961	0.961	0.964
	(75,75,50,50)	0.957	0.952	0.951	0.951	0.949	0.953	0.961	0.928	0.974	0.977	0.966	0.967	0.969	0.971
	(75,75,75,75)	0.973	0.954	0.939	0.973	0.929	0.954	0.939	0.879	0.968	0.975	0.957	0.960	0.964	0.968
	(100,100,100,100)	0.954	0.951	0.953	0.953	0.953	0.952	0.959	0.933	0.961	0.968	0.964	0.966	0.965	0.975
Normal 3	(20,20,20,20)	0.960	0.950	0.950	0.960	0.947	0.950	0.950	0.866	0.936	0.931	0.893	0.955	0.889	0.949
	(20,20,40,40)	0.949	0.955	0.957	0.949	0.953	0.955	0.957	0.898	0.929	0.920	0.848	0.957	0.853	0.944
	(30,30,30,30)	0.953	0.950	0.953	0.953	0.949	0.950	0.953	0.897	0.952	0.950	0.915	0.959	0.915	0.951
	(50,50,50,50)	0.930	0.955	0.953	0.954	0.956	0.953	0.953	0.926	0.948	0.946	0.930	0.958	0.931	0.952
	(75,75,50,50)	0.934	0.957	0.961	0.955	0.954	0.960	0.956	0.922	0.954	0.959	0.948	0.952	0.950	0.955
	(75,75,75,75)	0.930	0.948	0.951	0.950	0.948	0.951	0.955	0.907	0.958	0.958	0.947	0.957	0.945	0.960
	(100,100,100,100)	0.950	0.951	0.954	0.951	0.952	0.953	0.955	0.939	0.953	0.951	0.943	0.956	0.943	0.952
Normal 4	(20,20,20,20)	0.957	0.949	0.955	0.951	0.947	0.954	0.958	0.848	0.952	0.953	0.936	0.953	0.936	0.952
	(20,20,40,40)	0.965	0.954	0.957	0.955	0.954	0.955	0.948	0.877	0.942	0.939	0.923	0.957	0.924	0.947
	(30,30,30,30)	0.956	0.943	0.901	0.944	0.945	0.947	0.947	0.959	0.960	0.959	0.949	0.967	0.952	0.962
	(50,50,50,50)	0.965	0.958	0.962	0.944	0.945	0.947	0.947	0.964	0.961	0.958	0.953	0.958	0.954	0.963
	(75,75,50,50)	0.960	0.957	0.961	0.958	0.957	0.960	0.961	0.964	0.967	0.967	0.960	0.948	0.961	0.968
	(75,75,75,75)	0.953	0.948	0.953	0.947	0.946	0.949	0.953	0.936	0.962	0.964	0.958	0.953	0.958	0.963
	(100,100,100,100)	0.957	0.951	0.956	0.953	0.952	0.956	0.957	0.957	0.966	0.968	0.965	0.951	0.961	0.975
Normal 5	(20,20,20,20)	0.960	0.946	0.953	0.949	0.944	0.953	0.951	0.861	0.947	0.940	0.900	0.952	0.895	0.948
	(20,20,40,40)	0.969	0.952	0.956	0.954	0.952	0.954	0.950	0.876	0.941	0.931	0.895	0.954	0.892	0.946
	(30,30,30,30)	0.959	0.944	0.949	0.895	0.945	0.943	0.948	0.945	0.954	0.949	0.931	0.951	0.934	0.955
	(50,50,50,50)	0.948	0.963	0.954	0.954	0.953	0.957	0.957	0.916	0.963	0.959	0.947	0.956	0.945	0.960
	(75,75,50,50)	0.951	0.958	0.955	0.949	0.947	0.948	0.947	0.926	0.960	0.961	0.964	0.934	0.966	0.961
	(75,75,75,75)	0.942	0.957	0.943	0.955	0.953	0.960	0.953	0.942	0.966	0.964	0.959	0.945	0.959	0.970
	(100,100,100,100)	0.959	0.955	0.955	0.955	0.955	0.956	0.956	0.948	0.954	0.956	0.960	0.927	0.955	0.960
		Mean length													
Normal 1	(20,20,20,20)	1.402	0.864	1.523	0.727	0.650	0.895	1.821	0.880	1.208	1.442	1.019	1.208	1.112	1.803
	(20,20,40,40)	1.199	0.749	1.279	0.644	0.566	0.816	1.724	0.784	1.102	1.411	0.933	1.102	1.047	1.736
	(30,30,30,30)	1.193	0.715	1.241	0.602	0.532	0.756	1.627	0.790	1.065	1.391	0.903	1.065	1.009	1.714
	(50,50,50,50)	0.967	0.562	0.960	0.476	0.415	0.613	1.366	0.675	0.901	1.312	0.769	0.901	0.892	1.596
	(75,75,50,50)	0.918	0.517	0.875	0.433	0.378	0.559	1.266	0.659	0.854	1.293	0.726	0.854	0.851	1.559
	(75,75,75,75)	0.804	0.460	0.765	0.391	0.339	0.511	1.152	0.612	0.792	1.223	0.679	0.792	0.797	1.462
	(100,100,100,100)	0.693	0.400	0.617	0.339	0.294	0.446	0.997	0.570	0.712	1.167	0.611	0.712	0.733	1.375
Normal 2	(20,20,20,20)	0.604	0.604	0.604	0.604	0.604	0.604	0.604	0.776	1.017	1.206	0.853	1.017	0.932	1.532
	(20,20,40,40)	0.524	0.524	0.524	0.524	0.524	0.524	0.524	0.706	0.927	1.168	0.790	0.927	0.874	1.487
	(30,30,30,30)	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.684	0.899	1.161	0.756	0.899	0.843	1.471
	(50,50,50,50)	0.388	0.388	0.388	0.388	0.388	0.388	0.388	0.601	0.762	1.088	0.641	0.762	0.735	1.359
	(75,75,50,50)	0.354	0.354	0.354	0.354	0.354	0.354	0.354	0.544	0.662	1.008	0.602	0.719	0.701	1.327
	(75,75,75,75)	0.319	0.319	0.319	0.319	0.319	0.319	0.319	0.567	0.719	1.067	0.559	0.662	0.654	1.242
	(100,100,100,100)	0.277	0.277	0.277	0.277	0.277	0.277	0.277	0.514	0.602	0.948	0.507	0.602	0.599	1.170
Normal 3	(20,20,20,20)	1.114	0.747	0.884	0.551	0.537	0.574	1.190	1.445	1.044	1.003	0.846	1.044	0.864	1.320
	(20,20,40,40)	0.852	0.660	0.789	0.478	0.466	0.498	1.071	1.416	0.966	0.930	0.782	0.966	0.804	1.247
	(30,30,30,30)	0.820	0.615	0.665	0.438	0.429	0.454	0.999	1.411	0.931	0.903	0.754	0.931	0.774	1.215
	(50,50,50,50)	0.741	0.479	0.519	0.335	0.328	0.346	0.807	1.378	0.787	0.773	0.628	0.787	0.648	1.082
	(75,75,50,50)	0.707	0.434	0.468	0.302	0.296	0.312	0.744	1.358	0.735	0.725	0.578	0.735	0.599	1.012
	(75,75,75,75)	0.686	0.391	0.422	0.271	0.265	0.279	0.671	1.345	0.682	0.675	0.537	0.682	0.556	0.957
	(100,100,100,100)	0.658	0.340	0.366	0.234	0.229	0.241	0.591	1.332	0.622	0.618	0.481	0.622	0.502	0.877
Normal 4	(20,20,20,20)	1.180	0.723	0.961	0.576	0.555	0.612	1.101	2.269	1.069	1.030	0.877	1.069	0.904	1.343
	(20,20,40,40)	1.050	0.623	0.741	0.507	0.486	0.544	0.983	2.078	0.980	0.955	0.800	0.980	0.833	1.249
	(30,30,30,30)	1.012	0.584	0.662	0.460	0.445	0.487	0.895	2.217	0.948	0.916	0.800	0.980	0.833	1.249
	(50,50,50,50)	0.775	0.447	0.478	0.351	0.340	0.370	0.686	2.053	0.797	0.768	0.761	0.948	0.793	1.226
	(75,75,50,50)	0.604	0.361	0.370	0.315	0.307	0.330	0.623	2.080	0.758	0.726	0.605	0.758	0.633	0.998
	(75,75,75,75)	0.709	0.410	0.412	0.283	0.275	0.298	0.553	1.908	0.698	0.673	0.640	0.797	0.666	1.043
	(100,100,100,100)	0.491	0.312	0.318	1.791	0.245	0.237	0.257	0.476	0.641	0.617	0.502	0.641	0.527	0.866
Normal 5	(20,20,20,20)	1.886	0.760	0.794	0.581	0.567	0.606	1.139	2.201	1.099	1.041	0.890	1.099	0.911	1.353
	(20,20,40,40)	1.839	0.666	0.722	0.515	0.499	0.540	1.032	2.066	1.001	0.950	0.811	1.001	0.831	1.257
	(30,30,30,30)	1.668	0.619	0.641	0.464	0.454	0.482	0.943	2.217	0.963	0.918	0.773	0.963	0.795	1.225
	(50,50,50,50)	1.347	0.479	0.489	0.354	0.347	0.367	0.740	2.102	0.827	0.788	0.650	0.827	0.674	1.083
	(75,75,50,50)	1.162	0.436	0.433	0.317	0.311	0.326	0.673	2.154	0.774	0.733	0.614	0.774	0.636	1.024
	(75,75,75,75)	1.057	0.388	0.393	0.286	0.280	0.295	0.603	2.029	0.718	0.685	0.568	0.718	0.589	0.971
	(100,100,100,100)	0.860	0.337	0.338	0.247	0.242	0.255	0.525	1.884	0.647	0.618	0.507	0.647	0.527	0.882

Table 12. (Continued) Coverage probabilities (and mean length) of proposed 95% confidence intervals for optimal cut-points under normality (2000 simulations).

		GI							NPB						
		YI	MaxA	NWC	MMR	MinA	SEC	PYI	YI	MaxA	NWC	MMR	MinA	SEC	PYI
Scenario	Sample size	Coverage probability													
	(20,20,20,20)	0.972	0.949	0.951	0.947	0.948	0.949	0.955	0.866	0.936	0.931	0.891	0.949	0.894	0.948
	(20,20,40,40)	0.955	0.954	0.954	0.955	0.954	0.955	0.948	0.898	0.929	0.920	0.861	0.948	0.863	0.927
	(30,30,30,30)	0.961	0.944	0.943	0.944	0.945	0.947	0.947	0.897	0.952	0.950	0.929	0.955	0.928	0.946
Normal 6	(50,50,50,50)	0.960	0.955	0.954	0.954	0.955	0.952	0.952	0.926	0.948	0.946	0.943	0.964	0.945	0.957
	(75,75,50,50)	0.966	0.956	0.961	0.958	0.957	0.960	0.961	0.922	0.954	0.959	0.947	0.960	0.949	0.963
	(75,75,75,75)	0.956	0.946	0.949	0.947	0.946	0.949	0.953	0.907	0.958	0.958	0.949	0.964	0.952	0.961
	(100,100,100,100)	0.959	0.952	0.953	0.953	0.952	0.956	0.957	0.939	0.953	0.951	0.945	0.966	0.945	0.963
	(20,20,20,20)	0.968	0.956	0.937	0.948	0.947	0.950	0.955	0.926	0.953	0.946	0.917	0.935	0.916	0.891
	(20,20,40,40)	0.961	0.954	0.956	0.962	0.959	0.963	0.966	0.947	0.941	0.933	0.915	0.954	0.918	0.947
	(30,30,30,30)	0.956	0.948	0.949	0.937	0.936	0.941	0.946	0.956	0.954	0.949	0.866	0.925	0.865	0.869
Normal 7	(50,50,50,50)	0.959	0.961	0.957	0.955	0.954	0.957	0.956	0.955	0.956	0.954	0.929	0.949	0.930	0.908
	(75,75,50,50)	0.952	0.961	0.963	0.961	0.957	0.959	0.957	0.957	0.964	0.965	0.946	0.960	0.943	0.917
	(75,75,75,75)	0.954	0.948	0.950	0.945	0.943	0.944	0.965	0.962	0.966	0.968	0.934	0.959	0.931	0.913
	(100,100,100,100)	0.955	0.956	0.956	0.952	0.951	0.950	0.953	0.969	0.965	0.963	0.942	0.968	0.944	0.910
	(20,20,20,20)	0.956	0.949	0.950	0.946	0.947	0.949	0.953	0.974	0.953	0.952	0.915	0.955	0.916	0.961
	(20,20,40,40)	0.945	0.954	0.956	0.955	0.955	0.954	0.948	0.961	0.939	0.931	0.879	0.946	0.878	0.947
	(30,30,30,30)	0.951	0.946	0.945	0.943	0.945	0.944	0.948	0.964	0.947	0.945	0.927	0.950	0.928	0.949
Normal 8	(50,50,50,50)	0.948	0.955	0.956	0.954	0.953	0.956	0.955	0.972	0.963	0.960	0.956	0.969	0.957	0.970
	(75,75,50,50)	0.961	0.955	0.957	0.956	0.954	0.957	0.963	0.976	0.962	0.961	0.966	0.964	0.966	0.971
	(75,75,75,75)	0.951	0.947	0.944	0.944	0.944	0.944	0.954	0.972	0.964	0.959	0.953	0.964	0.953	0.967
	(100,100,100,100)	0.954	0.951	0.951	0.951	0.950	0.951	0.956	0.978	0.965	0.963	0.953	0.963	0.954	0.972
	(20,20,20,20)	0.960	0.950	0.950	0.949	0.950	0.949	0.957	0.956	0.938	0.929	0.825	0.944	0.825	0.946
	(20,20,40,40)	0.949	0.955	0.957	0.952	0.954	0.954	0.947	0.959	0.939	0.925	0.795	0.946	0.794	0.946
	(30,30,30,30)	0.953	0.950	0.953	0.950	0.950	0.951	0.949	0.964	0.943	0.936	0.885	0.949	0.886	0.955
Normal 9	(50,50,50,50)	0.953	0.956	0.955	0.956	0.957	0.957	0.955	0.970	0.959	0.957	0.934	0.961	0.936	0.966
	(75,75,50,50)	0.957	0.952	0.951	0.951	0.949	0.953	0.961	0.968	0.960	0.960	0.947	0.950	0.948	0.963
	(75,75,75,75)	0.973	0.954	0.939	0.973	0.929	0.954	0.939	0.971	0.961	0.956	0.947	0.952	0.949	0.966
	(100,100,100,100)	0.954	0.951	0.953	0.953	0.953	0.952	0.959	0.961	0.955	0.953	0.945	0.944	0.945	0.953
	(20,20,20,20)	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.927	0.915	0.907	0.878	0.915	0.881	0.921
	(20,20,40,40)	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.921	0.900	0.879	0.856	0.900	0.858	0.905
	(30,30,30,30)	0.957	0.957	0.957	0.957	0.957	0.957	0.957	0.950	0.935	0.931	0.924	0.935	0.923	0.941
Normal 10	(50,50,50,50)	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.967	0.957	0.947	0.949	0.957	0.949	0.963
	(75,75,50,50)	0.955	0.955	0.955	0.955	0.955	0.955	0.955	0.963	0.958	0.945	0.952	0.958	0.952	0.959
	(75,75,75,75)	0.953	0.953	0.953	0.953	0.953	0.953	0.953	0.968	0.964	0.958	0.959	0.964	0.960	0.966
	(100,100,100,100)	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.967	0.957	0.957	0.956	0.957	0.957	0.961
		Mean length													
	(20,20,20,20)	1.885	0.700	0.724	0.535	0.524	0.554	1.055	1.961	1.018	0.948	0.824	1.018	0.840	1.239
	(20,20,40,40)	1.837	0.613	0.632	0.465	0.455	0.481	0.942	1.964	0.925	0.858	0.756	0.925	0.772	1.150
	(30,30,30,30)	1.748	0.569	0.575	0.427	0.419	0.439	0.871	2.004	0.886	0.825	0.713	0.886	0.730	1.128
Normal 6	(50,50,50,50)	1.589	0.439	0.436	0.325	0.320	0.334	0.685	1.982	0.762	0.711	0.599	0.762	0.616	0.987
	(75,75,50,50)	1.484	0.400	0.391	0.294	0.290	0.301	0.627	1.936	0.710	0.656	0.556	0.710	0.572	0.927
	(75,75,75,75)	1.430	0.357	0.348	0.262	0.258	0.268	0.561	1.927	0.665	0.618	0.523	0.665	0.538	0.889
	(100,100,100,100)	1.316	0.310	0.300	0.226	0.223	0.231	0.490	1.817	0.603	0.560	0.471	0.603	0.486	0.806
	(20,20,20,20)	1.397	0.721	0.787	0.586	0.571	0.611	1.051	2.061	1.107	1.030	0.908	1.107	0.930	1.326
	(20,20,40,40)	1.304	0.618	0.663	0.510	0.495	0.535	0.917	2.009	0.943	1.119	0.816	1.022	0.837	1.245
	(30,30,30,30)	1.212	0.578	0.579	0.467	0.456	0.485	0.845	2.025	0.975	0.900	0.785	0.975	0.807	1.201
Normal 7	(50,50,50,50)	0.965	0.439	0.426	0.354	0.347	0.367	0.641	1.857	0.831	0.761	0.654	0.831	0.676	1.043
	(75,75,50,50)	0.855	0.400	0.378	0.319	0.313	0.329	0.583	1.781	0.781	0.714	0.616	0.781	0.638	0.978
	(75,75,75,75)	0.789	0.353	0.338	0.286	0.280	0.295	0.516	1.676	0.724	0.663	0.565	0.724	0.585	0.919
	(100,100,100,100)	0.675	0.305	0.290	0.247	0.242	0.255	0.446	1.592	0.663	0.606	0.511	0.663	0.530	0.855
	(20,20,20,20)	1.745	0.661	0.661	0.501	0.494	0.514	0.956	1.872	0.971	0.897	0.787	0.971	0.800	1.194
	(20,20,40,40)	1.652	0.581	0.575	0.431	0.426	0.441	0.844	1.864	0.920	0.843	0.737	0.920	0.753	1.110
	(30,30,30,30)	1.590	0.537	0.528	0.399	0.394	0.407	0.781	1.866	0.877	0.825	0.698	0.877	0.709	1.104
Normal 8	(50,50,50,50)	1.407	0.414	0.399	0.303	0.300	0.308	0.608	1.776	0.710	0.659	0.585	0.710	0.600	0.916
	(75,75,50,50)	1.315	0.375	0.360	0.275	0.272	0.279	0.555	1.715	0.680	0.626	0.531	0.680	0.545	0.851
	(75,75,75,75)	1.216	0.336	0.319	0.244	0.242	0.247	0.496	1.602	0.617	0.565	0.479	0.617	0.491	0.819
	(100,100,100,100)	1.315	0.291	0.275	0.211	0.209	0.213	0.431	1.529	0.582	0.532	0.450	0.582	0.465	0.761
	(20,20,20,20)	1.288	0.595	0.699	0.509	0.508	0.512	0.843	1.439	0.972	0.861	0.821	0.972	0.828	1.121
	(20,20,40,40)	1.118	0.572	0.515	0.440	0.439	0.441	0.741	1.351	0.889	0.779	0.745	0.889	0.751	1.019
	(30,30,30,30)	0.995	0.521	0.470	0.405	0.405	0.406	0.680	1.315	0.853	0.741	0.405	0.405	0.406	0.680
Normal 9	(50,50,50,50)	1.216	0.399	0.354	0.307	0.308	0.308	0.521	1.168	0.727	0.625	0.584	0.727	0.591	0.864
	(75,75,50,50)	1.042	0.359	0.317	0.277	0.278	0.277	0.473	1.114	0.684	0.585	0.497	0.633	0.502	0.767
	(75,75,75,75)	0.884	0.336	0.319	0.248	0.248	0.247	0.424	1.061	0.633	0.543	0.542	0.684	0.547	0.820
	(100,100,100,100)	0.744	0.281	0.244	0.214	0.214	0.214	0.370	0.989	0.577	0.491	0.449	0.577	0.455	0.706
	(20,20,20,20)	0.652	0.652	0.652	0.652	0.652	0.652	0.652	1.093	0.878	0.772	0.748	0.878	0.756	0.960
	(20,20,40,40)	0.563	0.563	0.563	0.478	0.466	0.498	1.071	1.014	0.814	0.693	0.685	0.814	0.692	0.885
	(30,30,30,30)	0.524	0.524	0.524	0.563	0.563	0.563	0.563	0.996	0.787	0.673	0.649	0.787	0.659	0.858
Normal 10	(50,50,50,50)	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.871	0.673	0.554	0.531	0.673	0.537	0.742
	(75,75,50,50)	0.363	0.363	0.363	0.323	0.323	0.323	0.323	0.818	0.630	0.520	0.498	0.630	0.502	0.694
	(75,75,75,75)	0.323	0.323	0.323	0.363	0.363	0.363	0.363	0.777	0.587	0.479	0.457	0.587	0.462	0.659
	(100,100,100,100)	0.279	0.279	0.279	0.279	0.279	0.279	0.279	0.725	0.539	0.433	0.409	0.539	0.413	0.605

Table 13. Coverage probabilities (and mean length) of proposed 95% confidence intervals for optimal cut-points under gamma distributions (2000 simulations).

		GammaGI1			GammaGI2			GammaGI3			NPB		
		YI	MaxA	NWC	YI	MaxA	NWC	YI	MaxA	NWC	YI	MaxA	NWC
Scenario	Sample size	Coverage probability											
	(20,20,20,20)	0.998	0.957	0.974	1.0000	0.946	0.958	0.999	0.942	0.979	0.955	0.951	0.948
	(20,20,40,40)	0.997	0.949	0.949	0.999	0.949	0.951	0.995	0.950	0.940	0.911	0.942	0.940
	(30,30,30,30)	0.997	0.953	0.963	0.995	0.946	0.959	0.987	0.965	0.975	0.954	0.954	0.958
Gamma 1	(50,50,50,50)	0.990	0.946	0.960	0.990	0.955	0.965	0.984	0.940	0.959	0.942	0.958	0.958
	(75,75,50,50)	0.990	0.953	0.971	0.987	0.950	0.971	0.983	0.956	0.968	0.962	0.966	0.970
	(75,75,75,75)	0.988	0.961	0.970	0.987	0.950	0.971	0.988	0.956	0.975	0.928	0.966	0.964
	(100,100,100,100)	0.982	0.959	0.980	0.978	0.949	0.970	0.980	0.946	0.969	0.958	0.964	0.964
	(20,20,20,20)	0.996	0.952	0.961	0.991	0.945	0.958	0.994	0.966	0.969	0.960	0.95	0.942
	(20,20,40,40)	0.989	0.961	0.963	0.986	0.944	0.955	0.990	0.956	0.965	0.948	0.932	0.916
	(30,30,30,30)	0.987	0.950	0.960	0.990	0.947	0.954	0.989	0.963	0.966	0.967	0.954	0.943
Gamma 2	(50,50,50,50)	0.984	0.949	0.950	0.976	0.945	0.947	0.977	0.955	0.956	0.973	0.960	0.957
	(75,75,50,50)	0.981	0.947	0.948	0.964	0.932	0.932	0.965	0.954	0.952	0.967	0.966	0.962
	(75,75,75,75)	0.979	0.957	0.954	0.970	0.942	0.944	0.965	0.951	0.947	0.970	0.962	0.959
	(100,100,100,100)	0.973	0.955	0.951	0.968	0.945	0.944	0.960	0.948	0.948	0.966	0.961	0.959
	(20,20,20,20)	0.985	0.952	0.961	0.992	0.950	0.965	0.992	0.961	0.970	0.965	0.949	0.942
	(20,20,40,40)	0.98	0.947	0.957	0.986	0.943	0.956	0.993	0.955	0.972	0.959	0.936	0.930
	(30,30,30,30)	0.984	0.950	0.960	0.983	0.940	0.956	0.985	0.958	0.965	0.970	0.956	0.948
Gamma 3	(50,50,50,50)	0.977	0.948	0.955	0.983	0.958	0.963	0.970	0.959	0.965	0.973	0.964	0.955
	(75,75,50,50)	0.973	0.954	0.952	0.970	0.948	0.953	0.965	0.960	0.961	0.974	0.972	0.969
	(75,75,75,75)	0.966	0.944	0.951	0.970	0.953	0.958	0.965	0.950	0.956	0.969	0.962	0.962
	(100,100,100,100)	0.962	0.949	0.954	0.968	0.947	0.954	0.965	0.947	0.953	0.975	0.964	0.960
	(20,20,20,20)	0.984	0.932	0.946	0.988	0.941	0.956	0.989	0.957	0.971	0.944	0.935	0.924
	(20,20,40,40)	0.963	0.937	0.952	0.964	0.937	0.956	0.975	0.944	0.958	0.935	0.913	0.901
	(30,30,30,30)	0.978	0.947	0.959	0.978	0.938	0.954	0.984	0.956	0.966	0.956	0.949	0.939
Gamma 4	(50,50,50,50)	0.972	0.947	0.957	0.959	0.942	0.947	0.973	0.955	0.959	0.961	0.951	0.946
	(75,75,50,50)	0.969	0.950	0.955	0.956	0.945	0.948	0.965	0.957	0.961	0.971	0.969	0.967
	(75,75,75,75)	0.961	0.959	0.963	0.954	0.939	0.944	0.965	0.957	0.961	0.971	0.958	0.960
	(100,100,100,100)	0.971	0.949	0.951	0.952	0.946	0.948	0.963	0.947	0.946	0.969	0.964	0.962
	(20,20,20,20)	0.977	0.932	0.951	0.976	0.944	0.962	0.974	0.946	0.961	0.931	0.931	0.924
	(20,20,40,40)	0.968	0.949	0.962	0.969	0.947	0.961	0.969	0.945	0.962	0.910	0.915	0.905
	(30,30,30,30)	0.975	0.942	0.956	0.966	0.942	0.954	0.977	0.948	0.957	0.947	0.953	0.936
Gamma 5	(50,50,50,50)	0.970	0.940	0.943	0.964	0.948	0.950	0.957	0.944	0.951	0.952	0.957	0.950
	(75,75,50,50)	0.966	0.947	0.948	0.955	0.946	0.944	0.955	0.955	0.959	0.963	0.966	0.962
	(75,75,75,75)	0.966	0.947	0.948	0.953	0.952	0.953	0.963	0.954	0.955	0.967	0.968	0.961
	(100,100,100,100)	0.960	0.952	0.959	0.942	0.945	0.942	0.954	0.942	0.940	0.969	0.965	0.961
		Mean length											
	(20,20,20,20)	2.378	0.606	4.574	2.723	0.593	4.282	2.778	0.600	4.489	1.065	1.123	1.149
	(20,20,40,40)	2.096	0.538	4.231	2.392	0.523	3.996	2.610	0.534	3.776	3.245	1.015	1.061
	(30,30,30,30)	1.943	0.501	3.942	2.288	0.489	3.816	2.238	0.502	4.028	3.465	0.996	1.051
Gamma 1	(50,50,50,50)	1.545	0.386	3.090	1.953	0.369	1.515	1.733	0.388	3.056	4.097	0.865	0.954
	(75,75,50,50)	1.429	0.347	2.461	1.653	0.339	1.659	1.571	0.350	2.674	4.324	0.816	0.901
	(75,75,75,75)	1.299	0.315	2.196	1.574	0.307	1.315	1.754	0.309	1.380	4.563	0.750	0.848
	(100,100,100,100)	1.109	0.273	1.808	1.317	0.265	1.142	1.390	0.267	1.251	4.902	0.685	0.781
	(20,20,20,20)	2.920	0.765	1.113	3.098	0.741	0.989	3.396	0.787	1.036	5.160	1.534	1.374
	(20,20,40,40)	2.726	0.660	0.740	2.759	0.633	0.685	3.140	0.675	0.751	5.029	1.398	1.229
	(30,30,30,30)	2.393	0.616	0.680	2.593	0.601	0.639	2.629	0.626	0.677	5.346	1.381	1.217
Gamma 2	(50,50,50,50)	1.741	0.474	0.473	1.873	0.463	0.462	1.777	0.474	0.473	5.553	1.184	1.030
	(75,75,50,50)	1.526	0.430	0.427	1.563	0.424	0.421	1.473	0.430	0.426	5.608	1.110	0.963
	(75,75,75,75)	1.252	0.383	0.380	01.259	0.376	0.374	1.232	0.384	0.381	5.648	1.016	0.881
	(100,100,100,100)	0.997	0.332	0.329	0.996	0.325	0.323	0.899	0.329	0.326	5.590	0.923	0.799
	(20,20,20,20)	0.768	0.271	0.441	1.017	0.265	0.403	1.006	0.279	0.414	1.222	0.530	0.469
	(20,20,40,40)	0.569	0.225	0.352	0.866	0.218	0.329	0.841	0.228	0.348	1.306	0.471	0.413
	(30,30,30,30)	0.601	0.222	0.332	0.922	0.215	0.302	0.902	0.220	0.296	1.277	0.462	0.406
Gamma 3	(50,50,50,50)	0.435	0.170	0.238	0.795	0.165	0.230	0.755	0.169	0.221	1.346	0.398	0.344
	(75,75,50,50)	0.393	0.156	0.198	0.719	0.153	0.199	0.673	0.155	0.184	1.339	0.379	0.328
	(75,75,75,75)	0.325	0.137	0.178	0.615	0.134	0.168	0.582	0.136	0.158	1.336	0.345	0.298
	(100,100,100,100)	0.264	0.119	0.142	0.492	0.116	0.134	0.431	0.117	0.125	1.333	0.313	0.268
	(20,20,20,20)	1.848	0.673	1.297	2.059	0.662	1.212	1.526	0.561	0.890	1.999	1.319	1.159
	(20,20,40,40)	1.628	0.588	0.930	1.710	0.579	0.848	1.906	0.608	0.915	1.899	1.195	1.037
	(30,30,30,30)	1.488	0.545	0.933	1.548	0.540	0.847	1.526	0.561	0.890	1.859	1.182	1.016
Gamma 4	(50,50,50,50)	1.037	0.421	0.545	0.978	0.416	0.515	0.961	0.425	0.523	1.686	1.013	0.860
	(75,75,50,50)	0.883	0.380	0.441	0.836	0.377	0.436	0.821	0.384	0.432	1.640	0.968	0.816
	(75,75,75,75)	0.743	0.343	0.364	0.697	0.338	0.354	0.695	0.344	0.353	1.552	0.893	0.752
	(100,100,100,100)	0.599	0.296	0.293	0.563	0.292	0.290	0.564	0.296	0.294	1.426	0.810	0.680
	(20,20,20,20)	1.009	0.373	0.669	1.000	0.364	0.616	1.056	0.389	0.663	1.027	0.721	0.624
	(20,20,40,40)	0.844	0.327	0.474	0.836	0.321	0.439	0.904	0.337	0.477	0.976	0.653	0.557
	(30,30,30,30)	0.752	0.300	0.424	0.695	0.296	0.388	0.722	0.307	0.418	0.981	0.653	0.548
Gamma 5	(50,50,50,50)	0.479	0.230	0.243	0.456	0.229	0.237	0.449	0.233	0.242	0.868	0.554	0.461
	(75,75,50,50)	0.416	0.207	0.207	0.385	0.206	0.205	0.380	0.210	0.207	0.849	0.541	0.447
	(75,75,75,75)	0.353	0.187	0.186	0.339	0.186	0.185	0.331	0.188	0.185	0.788	0.494	0.404
	(100,100,100,100)	0.293	0.162	0.160	0.279	0.161	0.159	0.278	0.162	0.160	0.734	0.449	0.366

Table 13. (Continued) Coverage probabilities (and mean length) of proposed 95% confidence intervals for optimal cut-points under gamma distributions (2000 simulations).

		GammaGI1				GammaGI2				GammaGI3				NPB			
		MMR	MinA	SEC	PYI	MMR	MinA	SEC	PYI	MMR	MinA	SEC	PYI	MMR	MinA	SEC	PYI
Scenario	Sample size	Coverage probability															
Gamma 1	(20,20,20,20)	0.954	0.951	0.959	0.988	0.944	0.946	0.948	0.972	0.943	0.945	0.954	0.981	0.938	0.957	0.946	0.957
	(20,20,40,40)	0.948	0.946	0.953	0.971	0.945	0.946	0.952	0.970	0.965	0.970	0.975	0.935	0.902	0.951	0.909	0.950
	(30,30,30,30)	0.960	0.961	0.962	0.972	0.941	0.937	0.947	0.964	0.955	0.952	0.965	0.977	0.941	0.953	0.946	0.968
	(50,50,50,50)	0.950	0.949	0.953	0.961	0.950	0.945	0.950	0.965	0.942	0.943	0.944	0.970	0.951	0.952	0.952	0.965
	(75,75,50,50)	0.945	0.947	0.945	0.960	0.953	0.955	0.953	0.959	0.950	0.949	0.951	0.955	0.946	0.947	0.948	0.975
	(75,75,75,75)	0.946	0.946	0.948	0.970	0.933	0.934	0.935	0.955	0.946	0.948	0.948	0.960	0.961	0.954	0.962	0.970
	(100,100,100,100)	0.952	0.952	0.951	0.959	0.948	0.946	0.950	0.950	0.940	0.940	0.942	0.949	0.957	0.957	0.954	0.969
Gamma 2	(20,20,20,20)	0.946	0.947	0.948	0.969	0.947	0.948	0.944	0.959	0.954	0.953	0.957	0.974	0.925	0.953	0.928	0.960
	(20,20,40,40)	0.954	0.953	0.955	0.970	0.947	0.947	0.948	0.942	0.949	0.949	0.951	0.972	0.903	0.933	0.904	0.934
	(30,30,30,30)	0.950	0.951	0.952	0.963	0.950	0.950	0.947	0.947	0.960	0.960	0.959	0.963	0.940	0.953	0.946	0.964
	(50,50,50,50)	0.950	0.949	0.948	0.957	0.947	0.947	0.947	0.945	0.948	0.949	0.949	0.964	0.954	0.961	0.956	0.965
	(75,75,50,50)	0.953	0.952	0.951	0.959	0.928	0.932	0.927	0.941	0.950	0.951	0.952	0.952	0.965	0.968	0.966	0.967
	(75,75,75,75)	0.954	0.953	0.954	0.958	0.946	0.944	0.945	0.946	0.948	0.946	0.949	0.955	0.959	0.962	0.962	0.971
	(100,100,100,100)	0.949	0.949	0.949	0.958	0.941	0.939	0.940	0.942	0.950	0.948	0.948	0.941	0.959	0.963	0.959	0.964
Gamma 3	(20,20,20,20)	0.947	0.948	0.957	0.969	0.940	0.939	0.948	0.953	0.955	0.956	0.961	0.966	0.889	0.907	0.889	0.956
	(20,20,40,40)	0.947	0.947	0.948	0.955	0.931	0.932	0.932	0.943	0.946	0.946	0.949	0.971	0.865	0.878	0.867	0.943
	(30,30,30,30)	0.951	0.953	0.955	0.962	0.940	0.941	0.943	0.944	0.951	0.951	0.953	0.971	0.926	0.915	0.928	0.963
	(50,50,50,50)	0.951	0.950	0.950	0.965	0.953	0.953	0.953	0.950	0.949	0.948	0.949	0.957	0.940	0.915	0.941	0.963
	(75,75,50,50)	0.952	0.953	0.952	0.961	0.950	0.951	0.950	0.945	0.947	0.947	0.947	0.958	0.938	0.927	0.938	0.968
	(75,75,75,75)	0.949	0.949	0.949	0.949	0.956	0.956	0.956	0.948	0.956	0.957	0.956	0.960	0.944	0.907	0.945	0.970
	(100,100,100,100)	0.949	0.949	0.949	0.959	0.950	0.950	0.949	0.955	0.943	0.943	0.944	0.952	0.945	0.903	0.946	0.963
Gamma 4	(20,20,20,20)	0.940	0.940	0.940	0.954	0.949	0.949	0.946	0.956	0.953	0.953	0.953	0.969	0.904	0.943	0.906	0.943
	(20,20,40,40)	0.929	0.930	0.931	0.941	0.935	0.936	0.935	0.934	0.943	0.944	0.944	0.957	0.863	0.932	0.867	0.922
	(30,30,30,30)	0.945	0.946	0.944	0.951	0.943	0.943	0.943	0.949	0.947	0.946	0.948	0.967	0.932	0.953	0.937	0.956
	(50,50,50,50)	0.951	0.950	0.951	0.955	0.941	0.940	0.942	0.946	0.954	0.954	0.955	0.957	0.947	0.955	0.946	0.952
	(75,75,50,50)	0.951	0.952	0.951	0.958	0.946	0.945	0.946	0.951	0.959	0.960	0.960	0.964	0.954	0.963	0.958	0.968
	(75,75,75,75)	0.962	0.962	0.962	0.959	0.943	0.943	0.944	0.947	0.959	0.960	0.960	0.964	0.948	0.958	0.951	0.965
	(100,100,100,100)	0.947	0.947	0.947	0.960	0.949	0.947	0.950	0.945	0.946	0.945	0.946	0.955	0.955	0.959	0.957	0.966
Gamma 5	(20,20,20,20)	0.939	0.940	0.939	0.951	0.947	0.946	0.951	0.949	0.942	0.942	0.943	0.955	0.888	0.942	0.887	0.933
	(20,20,40,40)	0.947	0.947	0.947	0.946	0.946	0.945	0.946	0.949	0.945	0.945	0.946	0.952	0.874	0.942	0.874	0.913
	(30,30,30,30)	0.940	0.939	0.939	0.950	0.940	0.938	0.940	0.948	0.945	0.948	0.944	0.949	0.938	0.949	0.939	0.949
	(50,50,50,50)	0.944	0.946	0.944	0.947	0.953	0.951	0.951	0.953	0.944	0.945	0.944	0.945	0.950	0.948	0.951	0.958
	(75,75,50,50)	0.956	0.957	0.956	0.948	0.936	0.937	0.936	0.948	0.950	0.949	0.950	0.950	0.961	0.952	0.963	0.966
	(75,75,75,75)	0.951	0.951	0.951	0.948	0.955	0.953	0.955	0.950	0.954	0.953	0.954	0.962	0.953	0.950	0.954	0.963
	(100,100,100,100)	0.954	0.954	0.954	0.957	0.943	0.943	0.944	0.941	0.943	0.943	0.943	0.948	0.949	0.940	0.951	0.964
		Mean length															
Gamma 1	(20,20,20,20)	0.528	0.526	0.694	1.438	0.522	0.520	0.665	1.356	0.551	0.549	0.698	1.484	0.926	1.123	0.958	1.502
	(20,20,40,40)	0.456	0.455	0.528	1.106	0.450	0.449	0.490	1.029	0.468	0.469	0.523	1.240	0.814	1.015	0.844	1.399
	(30,30,30,30)	0.436	0.434	0.505	1.071	0.431	0.429	0.488	0.981	0.447	0.445	0.556	1.271	0.805	0.996	0.838	1.409
	(50,50,50,50)	0.336	0.335	0.348	0.680	0.332	0.333	0.333	0.578	0.342	0.341	0.366	0.790	0.683	0.865	0.717	1.279
	(75,75,50,50)	0.308	0.307	0.315	0.601	0.305	0.304	0.312	0.579	0.312	0.311	0.336	0.666	0.642	0.816	0.676	1.232
	(75,75,75,75)	0.275	0.275	0.278	0.531	0.272	0.272	0.272	0.490	0.274	0.274	0.275	0.500	0.592	0.750	0.625	1.157
	(100,100,100,100)	0.239	0.238	0.240	0.464	0.235	0.235	0.235	0.434	0.236	0.236	0.238	0.444	0.533	0.685	0.565	1.057
Gamma 2	(20,20,20,20)	0.759	0.761	0.768	0.945	0.741	0.744	0.740	0.866	0.782	0.784	0.782	0.922	1.238	1.534	1.266	1.787
	(20,20,40,40)	0.648	0.650	0.647	0.776	0.632	0.635	0.630	0.727	0.662	0.663	0.661	0.780	1.105	1.398	1.128	1.657
	(30,30,30,30)	0.613	0.616	0.612	0.723	0.601	0.603	0.600	0.698	0.621	0.622	0.620	0.725	1.092	1.381	1.122	1.650
	(50,50,50,50)	0.472	0.474	0.471	0.548	0.465	0.467	0.463	0.526	0.472	0.474	0.471	0.541	0.910	1.184	0.935	1.443
	(75,75,50,50)	0.431	0.433	0.429	0.496	0.425	0.427	0.424	0.479	0.429	0.431	0.428	0.487	0.861	1.110	0.882	1.358
	(75,75,75,75)	0.382	0.384	0.381	0.440	0.378	0.379	0.376	0.422	0.383	0.384	0.381	0.434	0.778	1.016	0.799	1.264
	(100,100,100,100)	0.331	0.333	0.330	0.380	0.327	0.328	0.325	0.365	0.329	0.330	0.328	0.370	0.699	0.923	0.718	1.161
Gamma 3	(20,20,20,20)	0.287	0.289	0.286	0.335	0.283	0.284	0.282	0.323	0.299	0.300	0.298	0.337	0.449	0.530	0.452	0.623
	(20,20,40,40)	0.240	0.240	0.239	0.278	0.237	0.237	0.236	0.264	0.246	0.247	0.246	0.275	0.393	0.471	0.397	0.568
	(30,30,30,30)	0.239	0.240	0.238	0.274	0.232	0.233	0.232	0.262	0.239	0.240	0.238	0.268	0.392	0.462	0.398	0.557
	(50,50,50,50)	0.185	0.186	0.184	0.211	0.181	0.182	0.180	0.201	0.184	0.185	0.184	0.206	0.336	0.398	0.340	0.496
	(75,75,50,50)	0.171	0.172	0.171	0.194	0.168	0.169	0.167	0.186	0.170	0.171	0.169	0.188	0.323	0.379	0.326	0.475
	(75,75,75,75)	0.151	0.151	0.150	0.171	0.148	0.149	0.147	0.163	0.150	0.150	0.149	0.166	0.286	0.345	0.290	0.432
	(100,100,100,100)	0.130	0.131	0.130	0.147	0.128	0.129	0.128	0.141	0.129	0.129	0.129	0.142	0.257	0.313	0.261	0.398
Gamma 4	(20,20,20,20)	0.686	0.689	0.683	0.778	0.676	0.679	0.674	0.759	0.571	0.573	0.570	0.636	1.079	1.319	1.095	1.512
	(20,20,40,40)	0.584	0.586	0.582	0.678	0.580	0.582	0.578	0.657	0.611	0.612	0.610	0.692	0.960	1.195	0.975	1.410
	(30,30,30,30)	0.553	0.555	0.551	0.629	0.549	0.552	0.547	0.615	0.571	0.573	0.570	0.636	0.952	1.182	0.971	1.384
	(50,50,50,50)	0.429	0.430	0.427	0.480	0.424	0.425	0.422	0.467	0.433	0.434	0.431	0.477	0.791	1.013	0.802	1.204
	(75,75,50,50)	0.390	0.392	0.388	0.430	0.386	0.388	0.384	0.422	0.393	0.395	0.391	0.428	0.750	0.968	0.763	1.160
	(75,75,75,75)	0.350	0.351	0.348	0.387	0.346	0.347	0.344	0.377	0.351	0.352	0.350	0.384	0.683	0.893	0.696	1.091
	(100,100,100,100)	0.303	0.304	0.302	0.333	0.299	0.300	0.298	0.324	0.302	0.303	0.301	0.329	0.614	0.810	0.625	0.993
Gamma 5	(20,20,20,20)	0.376	0.378	0.376	0.431	0.368	0.369	0.368	0.414	0.394	0.394	0.393	0.442	0.579	0.721	0.589	0.821
	(20,20,40,40)	0.326	0.326	0.326	0.377	0.322	0.322	0.323	0.366	0.341	0.340	0.342	0.387	0.521	0.653	0.528	0.754
	(30,30,30,30)	0.304	0.305	0.303	0.342	0.300	0.301	0.300	0.333	0.312	0.313	0.312	0.346	0.520	0.653	0.528	0.756
	(50,50,50,50)	0.235	0.236	0.234	0.258	0.232	0.233	0.232	0.254	0.237	0.238	0.237	0.259	0.431	0.554	0.438	0.656
	(75,75,50,50)	0.213	0.214	0.211	0.230	0.210	0.211	0.209	0.226	0.214	0.215	0.213	0.230	0.416	0.541	0.422	0.639
	(75,75,75,75)	0.192	0.192	0.191	0.208	0.190	0.190	0.189	0.205	0.192	0.193	0.192	0.207	0.375	0.494	0.382	0.595
	(100,100,100,100)	0.166	0.167	0.166	0.180	0.164	0.165	0.164	0.176	0.166	0.166	0.165	0.178	0.335	0.449	0.341	0.544

Table 13. (Continued) Coverage probabilities (and mean length) of proposed 95% confidence intervals for optimal cut-points under gamma distributions (2000 simulations).

		GammaGI1			GammaGI2			GammaGI3			NPB		
		YI	MaxA	NWC	YI	MaxA	NWC	YI	MaxA	NWC	YI	MaxA	NWC
Scenario	Sample size	Coverage probability											
	(20,20,20,20)	0.983	0.952	0.964	0.975	0.943	0.954	0.986	0.962	0.978	0.943	0.933	0.927
	(20,20,40,40)	0.978	0.950	0.959	0.966	0.942	0.946	0.973	0.947	0.964	0.931	0.918	0.908
	(30,30,30,30)	0.981	0.946	0.954	0.977	0.953	0.954	0.979	0.955	0.961	0.958	0.944	0.936
Gamma 6	(50,50,50,50)	0.967	0.956	0.959	0.966	0.948	0.954	0.970	0.947	0.953	0.960	0.955	0.946
	(75,75,50,50)	0.971	0.954	0.953	0.957	0.947	0.945	0.967	0.960	0.959	0.964	0.959	0.954
	(75,75,75,75)	0.962	0.951	0.956	0.955	0.949	0.951	0.971	0.958	0.954	0.970	0.964	0.960
	(100,100,100,100)	0.966	0.951	0.946	0.951	0.949	0.946	0.954	0.943	0.943	0.966	0.965	0.961
	(20,20,20,20)	0.980	0.942	0.955	0.985	0.952	0.960	0.990	0.962	0.959	0.939	0.928	0.913
	(20,20,40,40)	0.971	0.945	0.948	0.973	0.939	0.948	0.984	0.956	0.956	0.921	0.905	0.898
	(30,30,30,30)	0.975	0.949	0.956	0.975	0.952	0.962	0.979	0.964	0.962	0.947	0.940	0.935
Gamma 7	(50,50,50,50)	0.970	0.950	0.954	0.954	0.954	0.955	0.971	0.954	0.950	0.968	0.955	0.952
	(75,75,50,50)	0.970	0.947	0.953	0.960	0.933	0.931	0.959	0.953	0.950	0.966	0.961	0.955
	(75,75,75,75)	0.969	0.953	0.953	0.951	0.944	0.946	0.957	0.947	0.944	0.969	0.960	0.956
	(100,100,100,100)	0.966	0.950	0.951	0.950	0.948	0.942	0.955	0.947	0.951	0.960	0.960	0.955
	(20,20,20,20)	0.979	0.942	0.956	0.980	0.950	0.961	0.983	0.962	0.969	0.941	0.935	0.926
	(20,20,40,40)	0.959	0.951	0.960	0.965	0.944	0.949	0.970	0.949	0.960	0.918	0.918	0.914
	(30,30,30,30)	0.962	0.951	0.952	0.967	0.954	0.955	0.972	0.957	0.961	0.955	0.949	0.944
Gamma 8	(50,50,50,50)	0.964	0.952	0.949	0.956	0.943	0.947	0.966	0.946	0.947	0.960	0.957	0.954
	(75,75,50,50)	0.967	0.955	0.949	0.957	0.954	0.956	0.954	0.959	0.956	0.964	0.964	0.954
	(75,75,75,75)	0.967	0.955	0.949	0.948	0.938	0.938	0.963	0.956	0.952	0.964	0.964	0.957
	(100,100,100,100)	0.964	0.950	0.949	0.948	0.943	0.951	0.951	0.941	0.937	0.969	0.958	0.955
	(20,20,20,20)	0.970	0.956	0.953	0.966	0.938	0.954	0.982	0.950	0.954	0.959	0.942	0.930
	(20,20,40,40)	0.973	0.957	0.964	0.964	0.942	0.951	0.974	0.952	0.968	0.951	0.924	0.900
	(30,30,30,30)	0.966	0.947	0.955	0.962	0.944	0.952	0.978	0.958	0.953	0.957	0.940	0.936
Gamma 9	(50,50,50,50)	0.957	0.942	0.947	0.951	0.952	0.951	0.963	0.950	0.954	0.960	0.952	0.945
	(75,75,50,50)	0.958	0.953	0.951	0.953	0.952	0.949	0.952	0.943	0.943	0.965	0.958	0.950
	(75,75,75,75)	0.961	0.948	0.953	0.956	0.955	0.953	0.958	0.951	0.948	0.961	0.955	0.951
	(100,100,100,100)	0.950	0.941	0.947	0.956	0.946	0.945	0.959	0.954	0.953	0.964	0.970	0.961
	(20,20,20,20)	0.968	0.953	0.956	0.961	0.955	0.956	0.971	0.951	0.947	0.911	0.915	0.906
	(20,20,40,40)	0.959	0.955	0.960	0.954	0.946	0.956	0.967	0.951	0.947	0.899	0.901	0.885
	(30,30,30,30)	0.961	0.948	0.950	0.958	0.953	0.956	0.969	0.963	0.959	0.938	0.933	0.923
Gamma 10	(50,50,50,50)	0.953	0.948	0.950	0.947	0.942	0.943	0.960	0.946	0.947	0.949	0.955	0.949
	(75,75,50,50)	0.952	0.944	0.947	0.956	0.948	0.953	0.958	0.951	0.947	0.959	0.966	0.957
	(75,75,75,75)	0.952	0.947	0.951	0.944	0.945	0.949	0.958	0.947	0.945	0.960	0.953	0.941
	(100,100,100,100)	0.959	0.960	0.956	0.958	0.950	0.958	0.948	0.943	0.943	0.967	0.960	0.956
		Mean length											
	(20,20,20,20)	0.976	0.390	0.529	1.041	0.379	0.475	1.094	0.405	0.517	1.029	0.736	0.640
	(20,20,40,40)	0.810	0.345	0.396	0.858	0.334	0.369	0.879	0.350	0.393	0.966	0.674	0.577
	(30,30,30,30)	0.710	0.314	0.337	0.746	0.307	0.328	0.741	0.318	0.328	0.961	0.658	0.558
Gamma 6	(50,50,50,50)	0.466	0.242	0.239	0.454	0.237	0.235	0.447	0.242	0.239	0.847	0.562	0.468
	(75,75,50,50)	0.389	0.217	0.213	0.360	0.213	0.209	0.365	0.218	0.214	0.822	0.537	0.444
	(75,75,75,75)	0.339	0.197	0.194	0.321	0.193	0.190	0.322	0.196	0.193	0.776	0.497	0.408
	(100,100,100,100)	0.283	0.170	0.167	0.268	0.167	0.165	0.267	0.169	0.167	0.709	0.452	0.370
	(20,20,20,20)	1.707	0.688	0.855	1.722	0.673	0.798	1.921	0.708	0.831	1.868	1.337	1.164
	(20,20,40,40)	1.439	0.597	0.627	1.432	0.578	0.606	1.591	0.607	0.642	1.748	1.201	1.024
	(30,30,30,30)	1.213	0.556	0.586	1.236	0.545	0.553	1.199	0.562	0.578	1.739	1.193	1.012
Gamma 7	(50,50,50,50)	0.779	0.425	0.429	0.732	0.418	0.423	0.727	0.426	0.428	1.557	1.029	0.858
	(75,75,50,50)	0.680	0.385	0.388	0.641	0.381	0.384	0.623	0.385	0.386	1.531	0.993	0.817
	(75,75,75,75)	0.568	0.344	0.347	0.527	0.340	0.344	0.539	0.344	0.347	1.414	0.911	0.751
	(100,100,100,100)	0.473	0.298	0.301	0.446	0.293	0.297	0.442	0.296	0.298	1.336	0.829	0.681
	(20,20,20,20)	1.012	0.445	0.604	1.349	0.440	0.546	1.437	0.463	0.596	1.148	0.847	0.730
	(20,20,40,40)	0.784	0.386	0.423	1.029	0.379	0.416	1.138	0.394	0.440	1.067	0.753	0.634
	(30,30,30,30)	0.726	0.359	0.385	0.854	0.354	0.371	0.857	0.365	0.379	1.079	0.755	0.632
Gamma 8	(50,50,50,50)	0.475	0.276	0.271	0.488	0.273	0.268	0.475	0.278	0.273	0.972	0.659	0.544
	(75,75,50,50)	0.419	0.251	0.244	0.409	0.249	0.241	0.399	0.252	0.244	0.925	0.622	0.504
	(75,75,75,75)	0.364	0.225	0.221	0.353	0.222	0.218	0.345	0.224	0.220	0.865	0.576	0.464
	(100,100,100,100)	0.306	0.195	0.191	0.294	0.192	0.189	0.291	0.193	0.190	0.798	0.519	0.418
	(20,20,20,20)	1.163	0.719	1.107	1.849	0.711	0.996	1.882	0.753	1.052	1.792	1.379	1.216
	(20,20,40,40)	1.008	0.634	0.735	1.280	0.618	0.692	1.398	0.650	0.727	1.713	1.281	1.105
	(30,30,30,30)	0.900	0.588	0.722	1.238	0.578	0.657	1.270	0.601	0.703	1.627	1.231	1.061
Gamma 9	(50,50,50,50)	0.618	0.452	0.462	0.672	0.446	0.458	0.687	0.458	0.465	1.426	1.040	0.875
	(75,75,50,50)	0.548	0.409	0.418	0.571	0.404	0.414	0.587	0.413	0.424	1.334	0.980	0.828
	(75,75,75,75)	0.485	0.369	0.373	0.475	0.363	0.368	0.485	0.369	0.374	1.275	0.929	0.775
	(100,100,100,100)	0.412	0.319	0.322	0.398	0.314	0.319	0.408	0.319	0.323	1.183	0.849	0.701
	(20,20,20,20)	0.492	0.383	0.409	0.640	0.379	0.396	0.641	0.394	0.406	0.818	0.690	0.593
	(20,20,40,40)	0.413	0.334	0.347	0.465	0.330	0.343	0.478	0.340	0.348	0.753	0.631	0.530
	(30,30,30,30)	0.372	0.310	0.321	0.403	0.305	0.315	0.405	0.314	0.320	0.761	0.626	0.525
Gamma 10	(50,50,50,50)	0.274	0.238	0.246	0.268	0.236	0.243	0.270	0.238	0.244	0.665	0.540	0.437
	(75,75,50,50)	0.241	0.216	0.223	0.236	0.213	0.218	0.236	0.215	0.220	0.644	0.522	0.415
	(75,75,75,75)	0.217	0.194	0.200	0.212	0.191	0.197	0.214	0.193	0.198	0.602	0.475	0.373
	(100,100,100,100)	0.186	0.168	0.173	0.181	0.165	0.171	0.181	0.166	0.170	0.556	0.436	0.339

Table 13. (Continued) Coverage probabilities (and mean length) of proposed 95% confidence intervals for optimal cut-points under gamma distributions (2000 simulations).

		GammaGI1				**GammaGI2**				**GammaGI3**				**NPB**			
		MMR	MinA	SEC	PYI	MMR	MinA	SEC	PYI	MMR	MinA	SEC	PYI	MMR	MinA	SEC	PYI
Scenario	Sample size	Coverage probability															
Gamma 6	(20,20,20,20)	0.948	0.948	0.948	0.954	0.939	0.940	0.940	0.949	0.955	0.955	0.955	0.955	0.757	0.943	0.757	0.938
	(20,20,40,40)	0.953	0.952	0.955	0.957	0.941	0.940	0.940	0.946	0.948	0.948	0.947	0.954	0.764	0.940	0.764	0.924
	(30,30,30,30)	0.950	0.951	0.951	0.952	0.952	0.952	0.951	0.955	0.945	0.946	0.945	0.957	0.867	0.944	0.869	0.950
	(50,50,50,50)	0.957	0.955	0.957	0.953	0.952	0.952	0.951	0.945	0.946	0.946	0.946	0.947	0.920	0.930	0.920	0.959
	(75,75,50,50)	0.954	0.954	0.953	0.956	0.939	0.938	0.940	0.950	0.951	0.951	0.951	0.961	0.945	0.914	0.946	0.963
	(75,75,75,75)	0.952	0.952	0.952	0.951	0.950	0.951	0.949	0.949	0.953	0.952	0.954	0.953	0.941	0.920	0.942	0.968
	(100,100,100,100)	0.949	0.949	0.950	0.954	0.947	0.947	0.947	0.953	0.939	0.940	0.939	0.946	0.944	0.902	0.945	0.965
Gamma 7	(20,20,20,20)	0.948	0.948	0.946	0.953	0.948	0.948	0.952	0.964	0.950	0.949	0.950	0.969	0.905	0.935	0.908	0.935
	(20,20,40,40)	0.938	0.938	0.940	0.946	0.935	0.935	0.934	0.944	0.949	0.948	0.949	0.965	0.884	0.916	0.886	0.906
	(30,30,30,30)	0.946	0.947	0.947	0.959	0.967	0.967	0.967	0.960	0.958	0.956	0.958	0.963	0.935	0.947	0.938	0.943
	(50,50,50,50)	0.949	0.950	0.950	0.956	0.955	0.955	0.954	0.947	0.947	0.947	0.948	0.953	0.950	0.956	0.952	0.962
	(75,75,50,50)	0.951	0.951	0.951	0.9509	0.938	0.938	0.938	0.946	0.949	0.947	0.949	0.952	0.960	0.959	0.961	0.964
	(75,75,75,75)	0.953	0.952	0.953	0.954	0.946	0.943	0.945	0.948	0.944	0.944	0.945	0.954	0.950	0.962	0.951	0.966
	(100,100,100,100)	0.949	0.949	0.950	0.956	0.944	0.944	0.943	0.943	0.952	0.950	0.952	0.950	0.957	0.958	0.959	0.957
Gamma 8	(20,20,20,20)	0.944	0.944	0.945	0.955	0.945	0.946	0.946	0.946	0.956	0.956	0.956	0.966	0.855	0.938	0.856	0.936
	(20,20,40,40)	0.943	0.943	0.944	0.953	0.944	0.944	0.944	0.945	0.945	0.945	0.945	0.949	0.834	0.922	0.833	0.908
	(30,30,30,30)	0.946	0.947	0.946	0.948	0.951	0.952	0.951	0.949	0.947	0.948	0.948	0.962	0.927	0.949	0.928	0.949
	(50,50,50,50)	0.950	0.950	0.951	0.947	0.942	0.942	0.941	0.943	0.948	0.948	0.948	0.956	0.952	0.959	0.952	0.958
	(75,75,50,50)	0.945	0.944	0.944	0.953	0.951	0.952	0.950	0.957	0.951	0.952	0.951	0.952	0.942	0.967	0.943	0.962
	(75,75,75,75)	0.954	0.953	0.954	0.958	0.944	0.944	0.944	0.939	0.956	0.956	0.956	0.960	0.955	0.963	0.956	0.963
	(100,100,100,100)	0.954	0.955	0.955	0.958	0.948	0.948	0.948	0.949	0.939	0.939	0.938	0.944	0.953	0.960	0.954	0.969
Gamma 9	(20,20,20,20)	0.948	0.948	0.943	0.960	0.941	0.941	0.940	0.942	0.960	0.960	0.960	0.967	0.690	0.953	0.691	0.950
	(20,20,40,40)	0.953	0.952	0.951	0.959	0.938	0.939	0.938	0.946	0.953	0.953	0.952	0.959	0.678	0.947	0.677	0.934
	(30,30,30,30)	0.956	0.957	0.956	0.956	0.956	0.956	0.954	0.950	0.947	0.947	0.948	0.957	0.811	0.950	0.812	0.954
	(50,50,50,50)	0.945	0.946	0.945	0.947	0.942	0.941	0.941	0.947	0.946	0.945	0.946	0.956	0.908	0.948	0.905	0.957
	(75,75,50,50)	0.950	0.950	0.949	0.953	0.951	0.951	0.951	0.953	0.948	0.948	0.948	0.942	0.943	0.950	0.942	0.971
	(75,75,75,75)	0.952	0.952	0.952	0.954	0.947	0.947	0.947	0.952	0.944	0.945	0.944	0.949	0.926	0.944	0.926	0.960
	(100,100,100,100)	0.949	0.949	0.949	0.948	0.949	0.948	0.949	0.951	0.951	0.951	0.950	0.951	0.918	0.960	0.920	0.971
Gamma 10	(20,20,20,20)	0.956	0.957	0.955	0.961	0.949	0.949	0.949	0.956	0.951	0.952	0.951	0.959	0.710	0.917	0.710	0.911
	(20,20,40,40)	0.954	0.954	0.953	0.947	0.945	0.944	0.945	0.943	0.947	0.947	0.946	0.953	0.718	0.902	0.719	0.898
	(30,30,30,30)	0.955	0.956	0.955	0.9480	0.945	0.945	0.945	0.948	0.952	0.951	0.952	0.964	0.862	0.934	0.862	0.931
	(50,50,50,50)	0.949	0.949	0.948	0.948	0.950	0.950	0.950	0.949	0.946	0.947	0.946	0.952	0.937	0.955	0.937	0.945
	(75,75,50,50)	0.949	0.949	0.948	0.946	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.956	0.943	0.965	0.944	0.964
	(75,75,75,75)	0.946	0.946	0.948	0.946	0.953	0.953	0.953	0.949	0.954	0.954	0.954	0.952	0.930	0.953	0.929	0.953
	(100,100,100,100)	0.954	0.954	0.954	0.955	0.956	0.956	0.956	0.952	0.949	0.949	0.950	0.952	0.940	0.963	0.939	0.963
		Mean length															
Gamma 6	(20,20,20,20)	0.382	0.383	0.381	0.441	0.373	0.374	0.372	0.422	0.394	0.395	0.393	0.453	0.519	0.736	0.524	0.822
	(20,20,40,40)	0.335	0.335	0.335	0.384	0.330	0.330	0.330	0.370	0.340	0.340	0.340	0.385	0.483	0.674	0.487	0.754
	(30,30,30,30)	0.309	0.310	0.308	0.351	0.304	0.305	0.303	0.340	0.312	0.313	0.312	0.351	0.479	0.658	0.485	0.742
	(50,50,50,50)	0.240	0.240	0.239	0.268	0.235	0.236	0.235	0.261	0.238	0.239	0.238	0.265	0.406	0.562	0.410	0.645
	(75,75,50,50)	0.215	0.216	0.214	0.238	0.211	0.212	0.210	0.231	0.214	0.215	0.213	0.236	0.391	0.537	0.394	0.623
	(75,75,75,75)	0.195	0.195	0.194	0.216	0.192	0.192	0.191	0.210	0.193	0.193	0.192	0.213	0.357	0.497	0.361	0.581
	(100,100,100,100)	0.168	0.169	0.168	0.186	0.166	0.167	0.166	0.182	0.167	0.167	0.166	0.183	0.319	0.452	0.323	0.530
Gamma 7	(20,20,20,20)	0.703	0.706	0.700	0.764	0.690	0.693	0.688	0.731	0.722	0.724	0.720	0.782	1.081	1.337	1.103	1.495
	(20,20,40,40)	0.604	0.606	0.603	0.654	0.593	0.595	0.591	0.621	0.613	0.614	0.613	0.664	0.949	1.201	0.963	1.356
	(30,30,30,30)	0.569	0.571	0.566	0.606	0.559	0.561	0.557	0.587	0.573	0.575	0.572	0.610	0.947	1.193	0.963	1.355
	(50,50,50,50)	0.436	0.438	0.434	0.457	0.430	0.432	0.429	0.443	0.436	0.438	0.435	0.455	0.799	1.029	0.811	1.194
	(75,75,50,50)	0.398	0.400	0.395	0.410	0.392	0.394	0.391	0.401	0.396	0.397	0.394	0.408	0.759	0.993	0.771	1.163
	(75,75,75,75)	0.354	0.355	0.352	0.366	0.350	0.351	0.348	0.356	0.353	0.354	0.352	0.365	0.690	0.911	0.701	1.073
	(100,100,100,100)	0.307	0.308	0.305	0.316	0.303	0.304	0.301	0.308	0.304	0.305	0.303	0.312	0.622	0.829	0.634	0.983
Gamma 8	(20,20,20,20)	0.486	0.488	0.485	0.527	0.478	0.479	0.477	0.512	0.505	0.506	0.504	0.541	0.701	0.847	0.708	0.967
	(20,20,40,40)	0.417	0.418	0.417	0.455	0.410	0.410	0.410	0.440	0.428	0.428	0.429	0.462	0.625	0.753	0.630	0.875
	(30,30,30,30)	0.395	0.396	0.394	0.418	0.386	0.387	0.385	0.405	0.401	0.401	0.400	0.420	0.635	0.755	0.645	0.875
	(50,50,50,50)	0.304	0.305	0.303	0.317	0.300	0.300	0.299	0.311	0.305	0.306	0.305	0.316	0.537	0.659	0.544	0.783
	(75,75,50,50)	0.278	0.280	0.277	0.285	0.273	0.274	0.271	0.279	0.277	0.278	0.276	0.281	0.506	0.622	0.510	0.735
	(75,75,75,75)	0.250	0.250	0.249	0.258	0.244	0.245	0.244	0.251	0.246	0.247	0.246	0.253	0.461	0.576	0.467	0.693
	(100,100,100,100)	0.216	0.216	0.215	0.222	0.211	0.212	0.211	0.217	0.213	0.214	0.213	0.218	0.413	0.519	0.419	0.635
Gamma 9	(20,20,20,20)	0.743	0.745	0.740	0.756	0.736	0.739	0.734	0.745	0.777	0.779	0.775	0.789	0.974	1.379	0.979	1.485
	(20,20,40,40)	0.650	0.653	0.648	0.657	0.642	0.644	0.640	0.639	0.666	0.667	0.664	0.666	0.940	1.281	0.944	1.369
	(30,30,30,30)	0.608	0.610	0.606	0.619	0.598	0.600	0.597	0.605	0.615	0.617	0.614	0.629	0.918	1.231	0.925	1.333
	(50,50,50,50)	0.467	0.468	0.466	0.473	0.462	0.463	0.460	0.465	0.471	0.472	0.469	0.477	0.769	1.040	0.775	1.144
	(75,75,50,50)	0.422	0.423	0.421	0.429	0.417	0.419	0.416	0.422	0.425	0.426	0.423	0.432	0.734	0.980	0.737	1.085
	(75,75,75,75)	0.381	0.382	0.380	0.385	0.377	0.378	0.376	0.377	0.380	0.381	0.379	0.383	0.675	0.929	0.679	1.032
	(100,100,100,100)	0.330	0.331	0.329	0.332	0.326	0.327	0.325	0.326	0.329	0.330	0.328	0.331	0.615	0.849	0.620	0.947
Gamma 10	(20,20,20,20)	0.382	0.383	0.381	0.441	0.432	0.432	0.431	0.402	0.450	0.451	0.449	0.420	0.522	0.690	0.526	0.746
	(20,20,40,40)	0.335	0.335	0.335	0.384	0.372	0.373	0.372	0.349	0.383	0.383	0.383	0.363	0.501	0.631	0.503	0.682
	(30,30,30,30)	0.309	0.310	0.308	0.351	0.349	0.349	0.348	0.320	0.358	0.359	0.358	0.330	0.507	0.626	0.513	0.678
	(50,50,50,50)	0.240	0.240	0.239	0.268	0.270	0.270	0.269	0.246	0.272	0.273	0.272	0.248	0.450	0.540	0.454	0.593
	(75,75,50,50)	0.215	0.216	0.214	0.238	0.245	0.245	0.244	0.218	0.247	0.248	0.247	0.221	0.432	0.522	0.434	0.579
	(75,75,75,75)	0.195	0.195	0.194	0.216	0.219	0.220	0.219	0.199	0.221	0.222	0.221	0.201	0.386	0.475	0.390	0.534
	(100,100,100,100)	0.168	0.169	0.168	0.186	0.190	0.190	0.190	0.172	0.190	0.191	0.190	0.172	0.344	0.436	0.347	0.487

Table 14. Optimal cut-point estimates and corresponding statistics for Ovarian cancer biomarker CA125. The confidence intervals were estimated by non-parametric bootstrap method.

Method	cut-point estimate	95% confidence interval	Sen_o	Spe_o	CCR_o
YI	35.290	(25.510, 108.300)	0.631	0.715	1.347
MaxA	35.290	(25.410, 64.707)	0.631	0.715	1.347
NWC	33.390	(25.620, 38.650)	0.657	0.687	1.344
MMR	35.290	(25.998, 52.710)	0.631	0.715	1.347
MinA	35.290	(25.410, 64.707)	0.631	0.715	1.347
SEC	35.290	(25.998, 52.710)	0.631	0.715	1.347
PYI	35.290	(25.410, 105.600)	0.631	0.715	1.347

Table 15. Optimal cut-point estimates and corresponding statistics at cut-points for Ovarian cancer biomarker IGF2. The confidence intervals were estimated by non-parametric bootstrap method.

Method	cut-point estimates	95% confidence interval	Sen_o	Spe_o	CCR_o
YI	2540.058	(1172.362, 2540.058)	0.001	1.000	1.001
MaxA	1713.162	(1636.493, 1835.562)	0.485	0.505	0.991
NWC	1713.162	(1636.493, 1835.314)	0.485	0.505	0.991
MMR	1706.937	(1608.197, 1746.806)	0.493	0.493	0.986
MinA	1706.937	(1636.493, 1835.562)	0.493	0.493	0.986
SEC	1706.937	(1608.197, 1746.806)	0.493	0.493	0.986
PYI	1713.162	(1469.202, 1845.237)	0.485	0.505	0.991

FIGURES

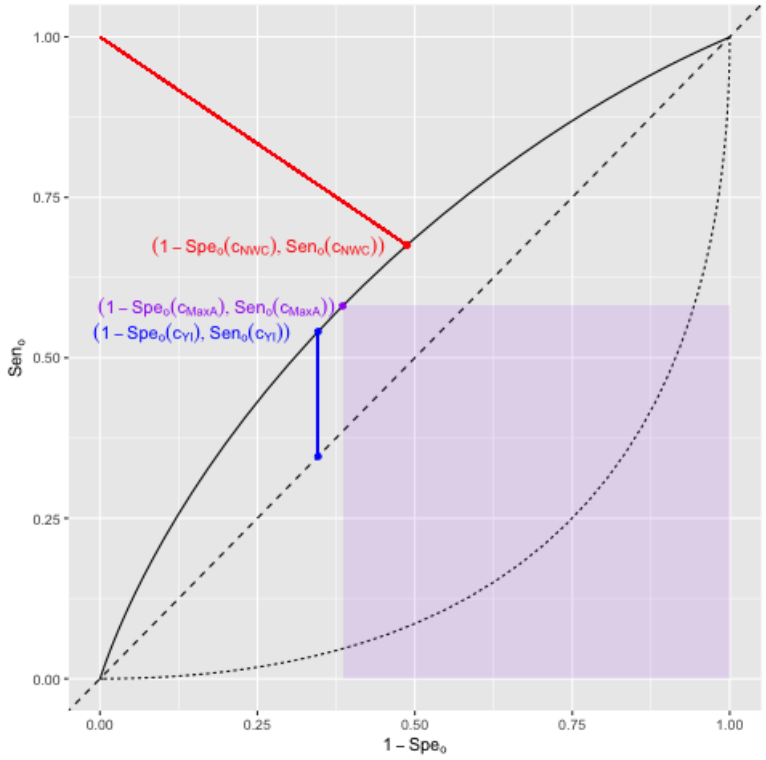

Figure 1 Illustration of the Youden Index , Max Area, and Northwest Corner methods for optimal cut-point selection when subclasses involved under binary classification, where $Y_1 \sim (0, 1)$, $Y_2 \sim (2.7, 0.95)$, $Y_3 \sim (3.2, 1)$, and $Y_4 \sim (5.26, 1)$. The dotted curve is the chance curve. The three points with coordinates $(1 - Spe_o(), Sen_o())$ correspond to three selection methods, and c_{YI}, c_{MaxA}, and c_{NWC} refer to the cut-points determined by Youden Index (blue), Max Area (purple), and Northwest Corner (red) methods, respectively.

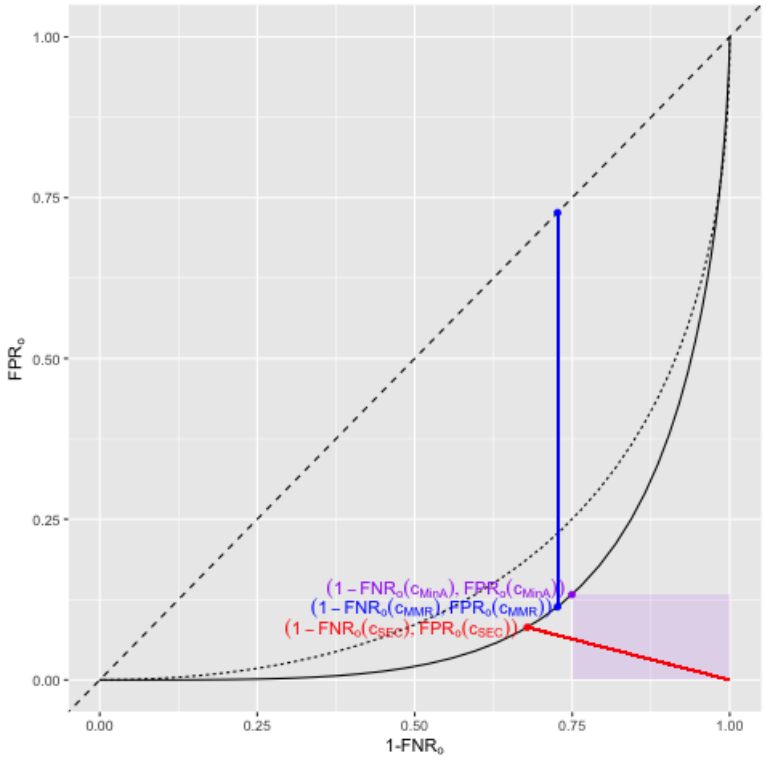

Figure 2 Illustration of the Overall Minimum Misclassification Rate, Min Area, and Southeast Corner methods for optimal cut-point selection when subclasses involved under binary classification, where $Y_1 \sim (1.0, 1.0)$, $Y_2 \sim (1.0, 1.0)$, $Y_3 \sim (1.2, 1.2)$, and $Y_4 \sim (1.5, 1.4)$. The dashed curve is the chance curve. The dotted curve is the chance curve. The three points with coordinates $(1-Spe_o(), Sen_o())$ correspond to three selection methods, and c_{MMR}, c_{MinA}, and c_{SEC} refer to the cut-points determined by Overall Minimum Misclassification Rate (blue), Min Area (purple), and Southeast Corner (red) methods, respectively.

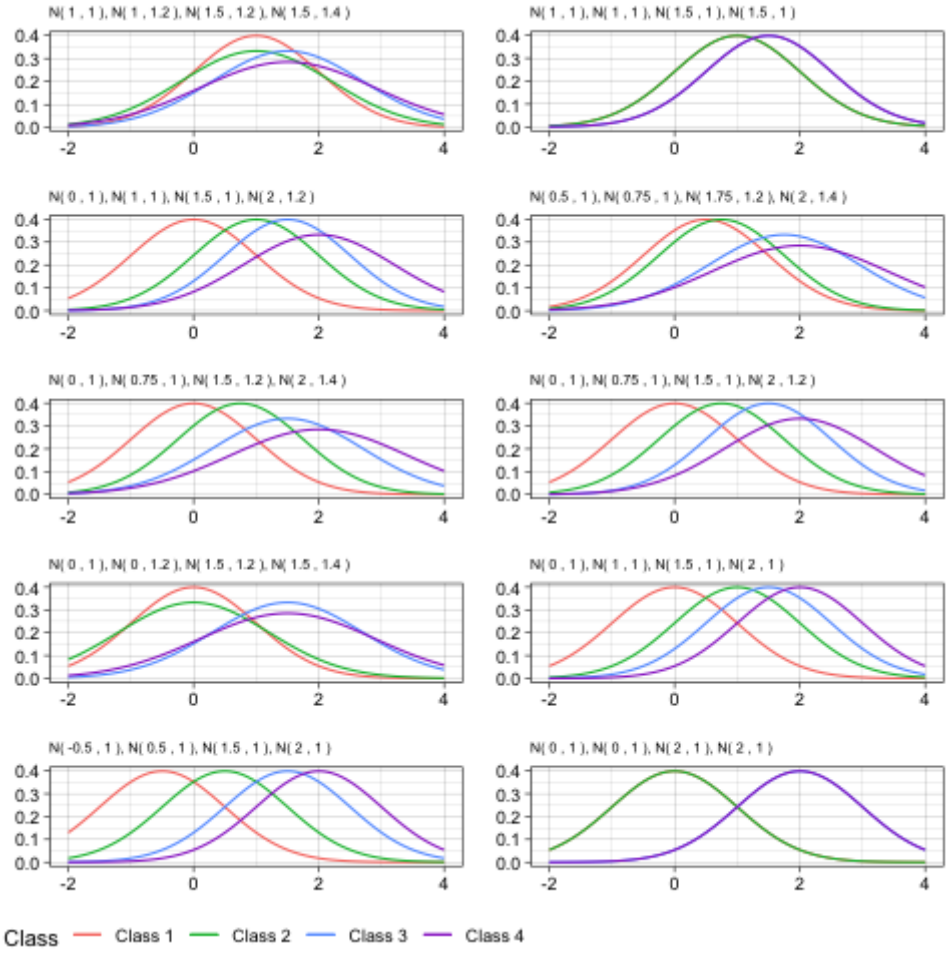

Figure 3 Density plots of hypothesized biomarkers under normality in simulation studies.

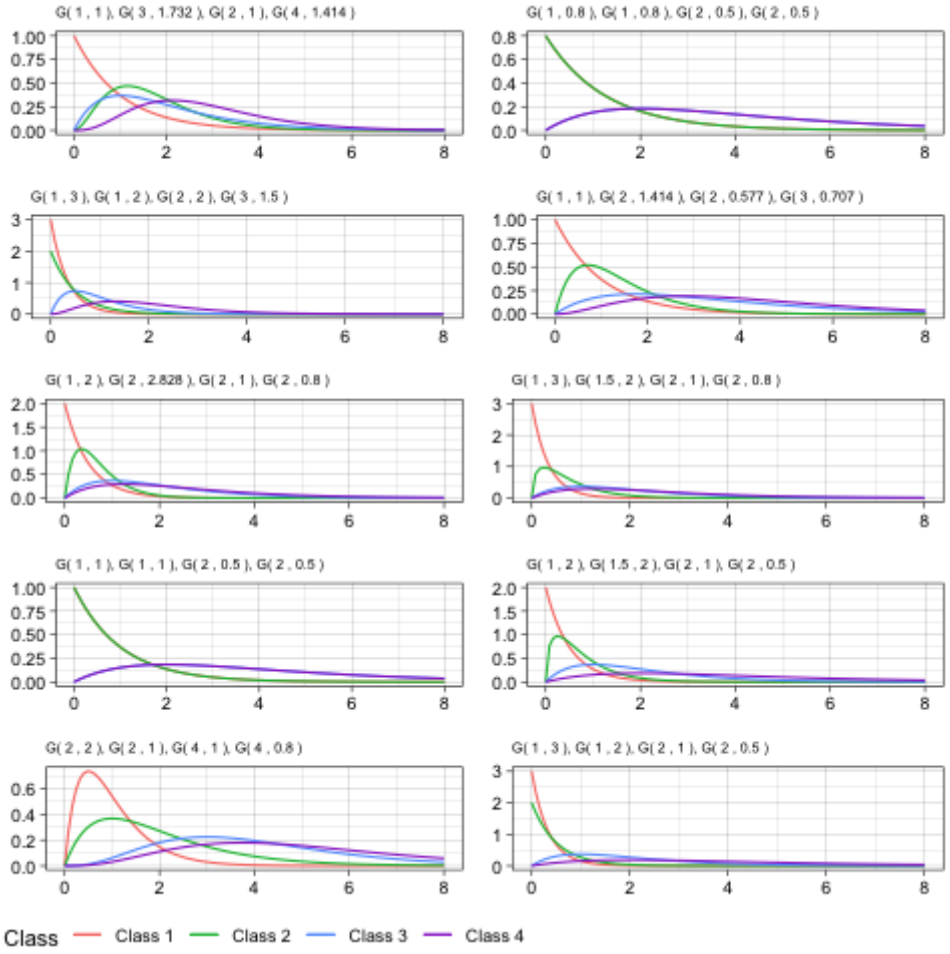

Figure 4 Density plots of hypothesized biomarkers under gamma distribution in simulation studies.

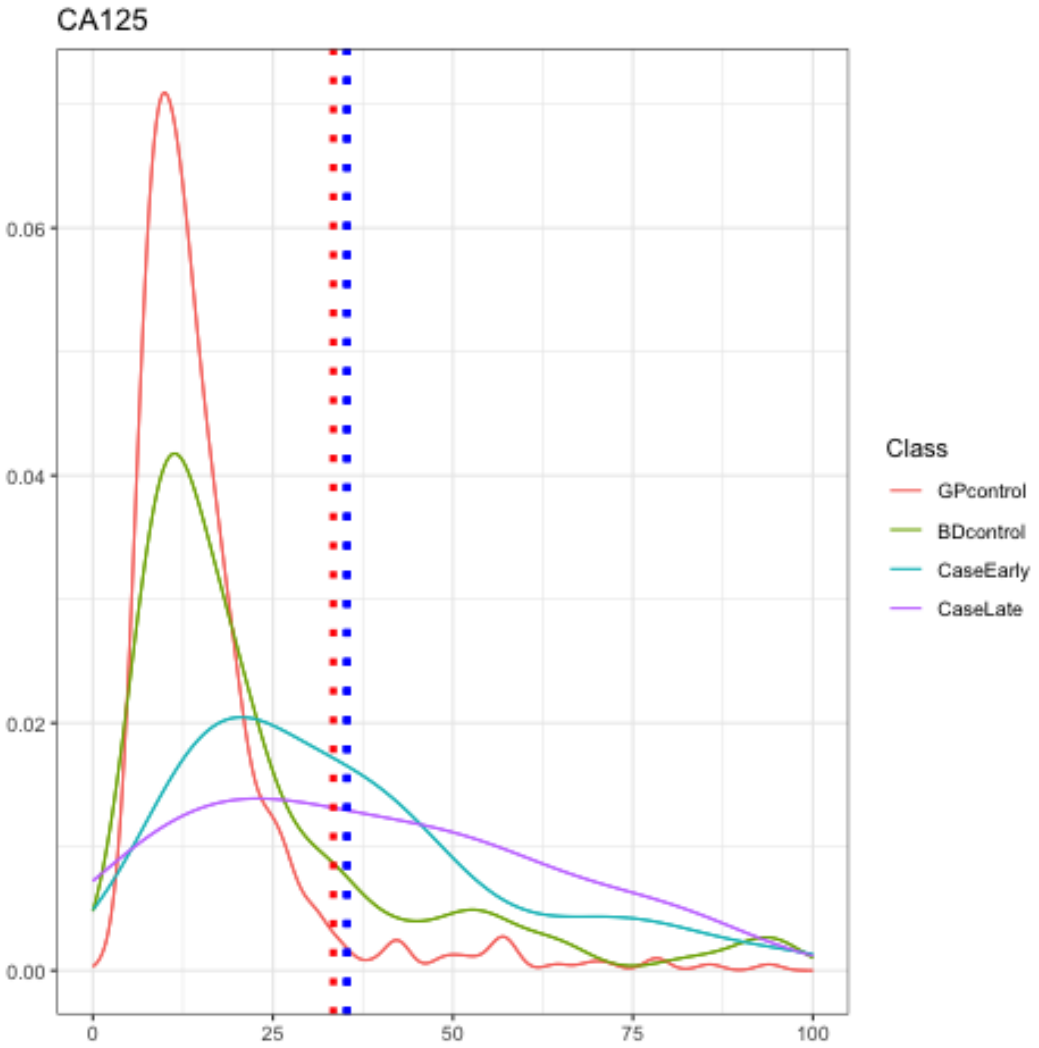

Figure 5 Estimated optimal cut-points for biomarker CA125: blue dotted line refers to cut-points by the YI, MaxA, MMR, MinA, SEC, and PYI methods; and green dotted line refers to cut-point by NWC method.

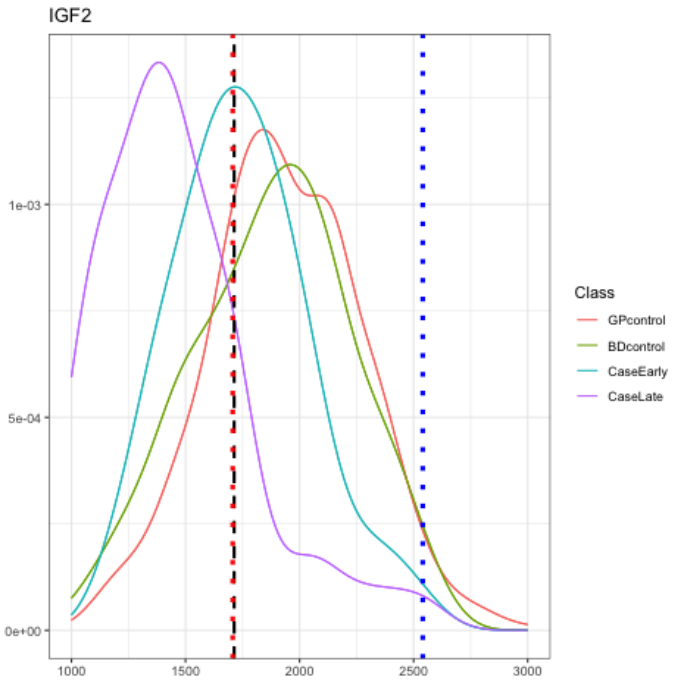

Figure 6 Estimated optimal cut-points for biomarker IGF2: blue dotted line refers to cut-point by the YI method; black dashed line refers to cut-points by the MaxA, NWC, and PYI methods; and red dotted line refers to cut-points by the MMR, MinA, and SEC methods.